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Systematic Error Correction in Determining the Total Phase in Integer Interferometry

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Abstract—A modified integer interferometry method is proposed for eliminating the interference fringe profile distortions caused by errors in interferogram recording and processing.

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INTRODUCTION

Development of modern technologies is impossible without improving precision measurement systems. The main element in laser interference systems is an interferometer that compares the object and reference wavefields [1]. The brightness field $I(x, y)$ arising when the reference and object optical fields interfere is described by

$$I(x, y) = A(x, y) + B(x, y) \cos(\Phi(x, y)), \tag{1}$$

where $A(x, y)$ is the average brightness, $B(x, y)$ is the fringe amplitude, and $\Phi(x, y)$ is the field of phase differences of the interfering optical fields (the total phase):

$$\Phi(x, y) = \phi(x, y) + 2\pi N(x, y). \tag{2}$$

Herein, $N(x, y)$ is the number of integer periods 2π fitted into the total optical difference $\Phi(x, y)$ and depending on the interferometer geometry and the laser wavelength λ, and $\phi(x, y)$ is the local phase that is a fraction of $\Phi(x, y)$. We will omit the coordinates (x, y) for simplicity.

Determining the integer number of periods N is known as elimination of phase ambiguity [2]. There are numerous algorithms realizing this approach [3, 4]. They are usually based on estimating the discontinuities of the local phases ϕ and choosing the way for their space integration in such a manner that the integration domain would not contain the discontinuities [3], or summing up the correction coefficients proportional to the number of discontinuities with regard to their sign [3]. It is evident that in the case of a discontinuity position error, the errors are accumulated, hence, the possible measurement range becomes limited. Moreover, the methods give no way of detecting a phase discontinuity exceeding 2π, which is the principal limitation of single-frequency interference systems. An integer interferometer approach is proposed in [5] for reconstructing the total phase Φ from values of local phases ϕ via several laser wavelengths. This approach is useful in preventing the limitations, but is highly sensitive to measurement errors of the local phases ϕ.

It is convenient to replace the interfering wave phase difference Φ by a proportional quantity Λ, that is, an optical path difference (OPD) (hereafter, path difference). The total path difference Λ is related to the total phase Φ as follows:

$$\Lambda = \frac{\lambda}{2\pi}\Phi = \frac{\lambda}{2\pi}(\phi + 2\pi N) = \delta + \lambda N, \tag{3}$$

where δ is the local path difference corresponding to the local phase ϕ (this quantity varies from zero to a period of interference fringe in wavelengths).

The measured values of ϕ are converted to local path differences δ expressed as integer values with the number of signs which ensure the necessary measurement accuracy.

For determining the OPD, it is required to find the solution to the integral simultaneous congruences

$$\begin{cases} \Lambda \equiv \delta_1 \bmod m_1, \\ \Lambda \equiv \delta_2 \bmod m_2. \end{cases} \tag{4}$$

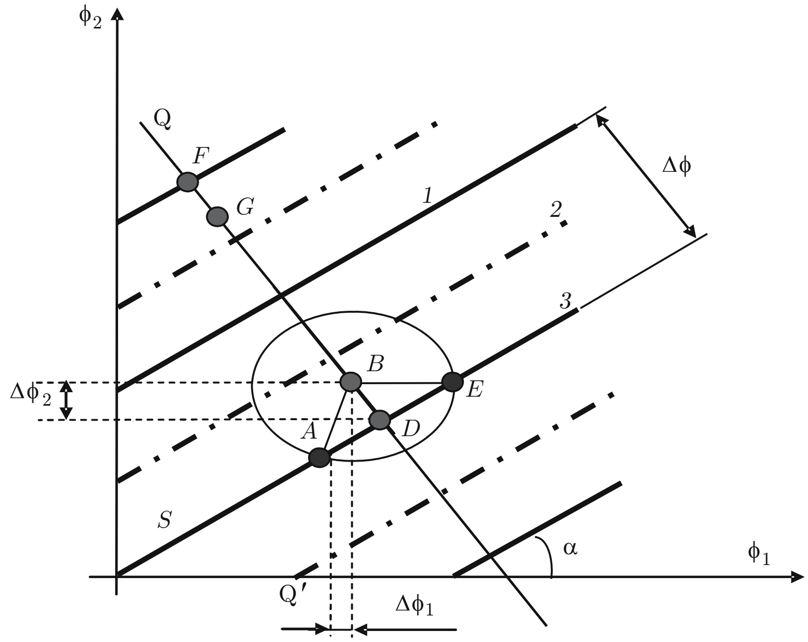

Fig. 1. The interference signal trajectory on the complex plane.

The solution may be represented as a trajectory on the complex plane (δ_1, δ_2) (Fig. 1). The maximal trajectory length $L_{\max} = m_1 \times m_2 - 1$ defines the dynamic range of unique determination of $\Lambda_{\max}$. The distance between the neighboring diagonals of the trajectory $\delta L = 1$. If $\Lambda < \Lambda_{\max}$, then $\delta L > 1$. For example, if $m_1 = 633$, $m_2 = 488$ (which corresponds to He–Ne and Ar lasing lines), and $L_{\max} = 10m_2$, then $\delta L = 52$.

In the case of inexact initial values $\delta^*_{1,2} = \delta_{1,2} + \varepsilon_{1,2}$, the solution to (4) leads to gross errors because the path difference Λ^* corresponding to δ^*_1 and δ^*_2 on the complex plane is greater than $\Lambda_{\max}$. We can show that if the complex plane point corresponding to δ^*_1 and δ^*_2 deviates from the nearest trajectory diagonal by no more than $\delta L/2$, the error can be compensated [5].

The major contribution to the desired error is from fringe profile deviation from the form of (1). Most frequently the deviation of this dependence from a cosine form is caused by a speckle image structure under coherent illumination [6].

Our goal is to modify the basic method in order to increase its robustness to measurement errors caused by using the fringe profile.

MODIFICATION OF THE BASIC METHOD

For eliminating the errors caused by a distorted fringe profile, it is necessary to carry out the following steps:

1. Calculate the difference of local path differences: $\Delta = \delta_1 - \delta_2$.
2. Determine isotropic regions, i.e., interferogram regions in which

$$\Lambda = \text{const.} \tag{5}$$

We should note that the errors of determining the isotropic region boundaries, which arise at some points and are caused by a small path difference SNR, are local and do not extend to the neighboring regions.

3. Estimate the difference in isotropic regions. In the absence of noises, the difference Δ is a piecewise constant function regardless of the law of changing OPD. Without loss of stability at the step of correction, the difference estimate may deviate by a value less than the half distance between diagonals 1 and 3 of signal line 2 (see Fig. 1).
4. Form the phase planes (Δ_1, δ_2), (Γ_1, δ_2), and (δ_1, Γ_2) by the rule:

$$\Gamma_1 = \delta_2 + \Delta_j; \qquad \Gamma_2 = \begin{cases} \delta_1 + \Delta_j & \text{if} \quad \delta_1 - \Delta_h \geqslant 0, \\ \delta_1 - \Delta_j + m_2 & \text{else.} \end{cases} \tag{6}$$

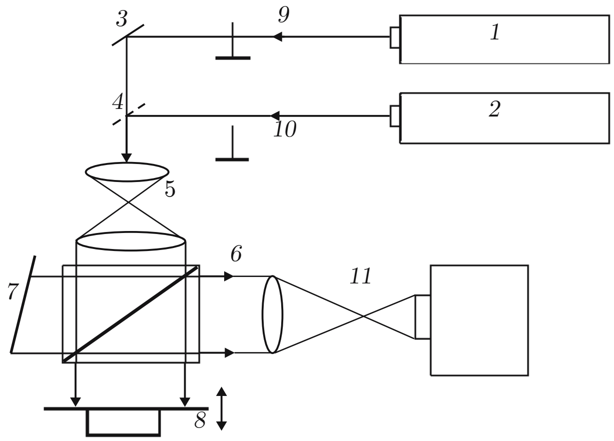

Fig. 2. An optical scheme of the interferometer: lasers *1* and *2*; mirrors *3* and *7*; half-transmitting mirror *4*; collimator *5*; beam splitting cube *6*; movable reference mirror *8*; beam choppers *9* and *10*; input device *11*.

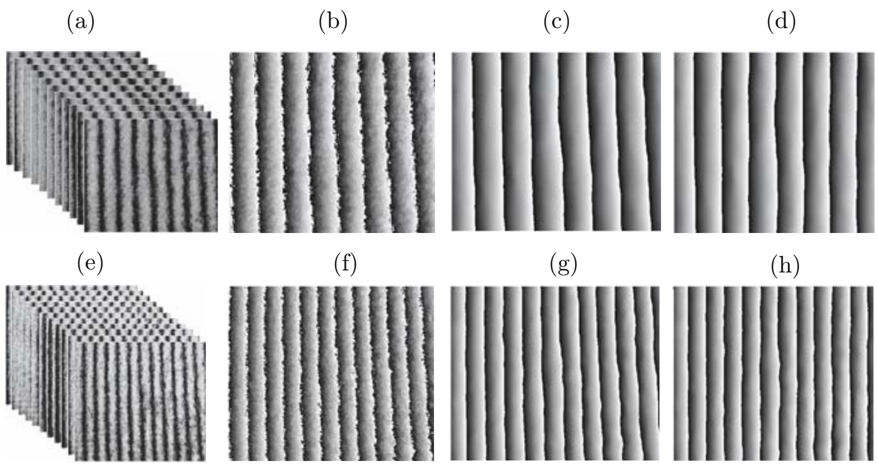

Fig. 3. Results for the first step of algorithm correction: interferograms (a and e), local phases (b and f), and the local phases after filtering for wavelengths of 633 nm (c and g) and 488 nm (d and h).

5. Correct the path differences on the phase plane and find the total path difference (OPD) by basic algorithm (4).

Experimental validation was carried out via measuring the reference object relief by the basic and proposed methods [1]. Figure 2 represents an optical scheme of the laser interferometric system.

At Step 1, we recorded several interferograms with different phase shifts. The phase shift introduced between interferogram exposures was $\lambda/10$. The interferograms were recorded as follows: at first we recorded interferograms with different wavelengths, then changed position of reference mirror 8. We recorded 22 interferograms: 11 items with a wavelength of 633 nm and 11 items with a wavelength of 488 nm (Figs. 3a and 3e). Figures 3b and 3f show results of calculating the local phases. For reducing the destabilizing factors, we carried out filtering of speckle noises [4] (Figs. 3c and 3g) and elimination of wave aberrations of the optical interferometer scheme [5] (Figs. 3d and 3h), respectively.

At Step 2, we calculated the local phases ϕ_1 and ϕ_2 according to the approach described in [3]. Figures 4a and 4b represent the interferogram phase profiles ϕ_1 and ϕ_2 for wavelengths of 633 and 488 nm, respectively, and the phase differences between them (Fig. 4c). Using the calculated phase difference $\Delta\phi$, we determined the isophase regions ($\Delta\phi = \text{const}$) and calculated the synthesized phases φ_1 and φ_2.

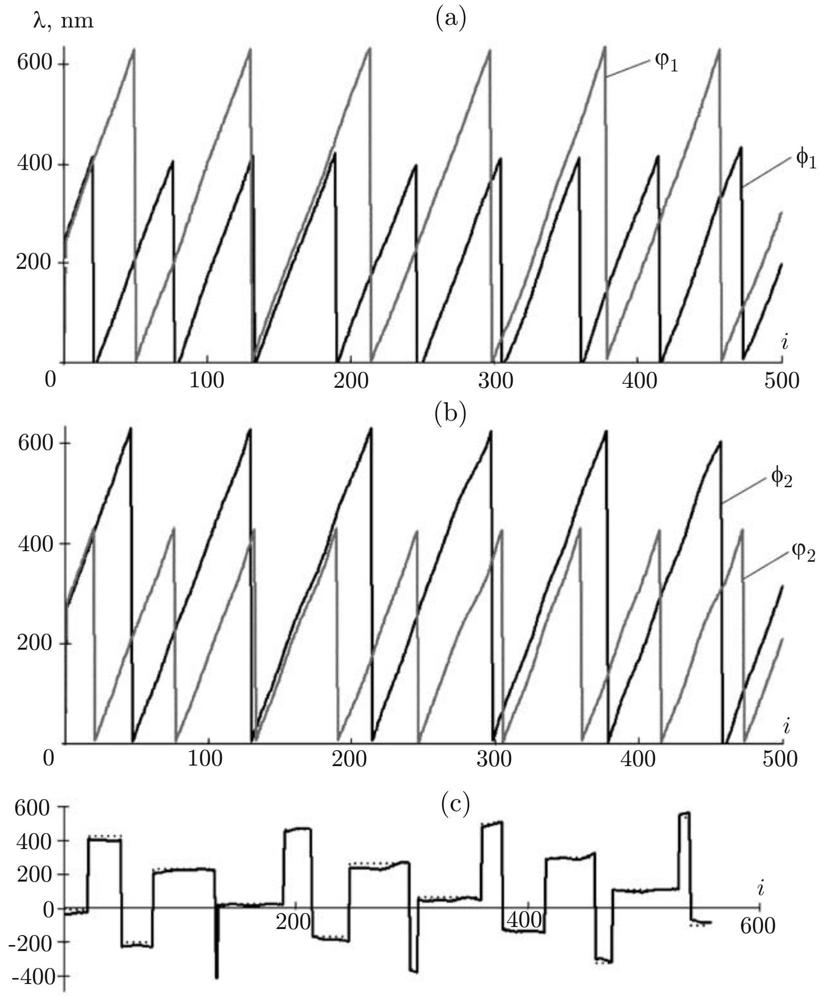

Fig. 4. Phase profiles at Step 2: (a and b) profiles of phases ϕ_1 and ϕ_2, and the corresponding synthesized phases φ_1 and φ_2; (c) phase difference $\Delta\phi$.

At step 3, we formed the phase planes (ϕ_1, ϕ_2) by the basic method, phase planes (ϕ_1, φ_2) and (φ_1, ϕ_2), and calculated the corresponding total phases Φ, Φ_1, and Φ_2 by the modified method (Figs. 5a–5f). Results of correcting the total phase trajectory to the nearest allowed diagonal of the signal line (by rules (4)-(6)) for the basic and modified methods are depicted in Figs. 5b, 5d, and 5f. Figure 5b shows that the resulting error of the total phase after eliminating the phase ambiguity for the basic method is much greater than the wavelength λ because of the gross errors caused by distortion of the interference fringe profile during phase correction. The measurement error of the total phase after eliminating the phase ambiguity by the modified method does not exceed the measurement error of local phases, as Figs. 5d and 5f show.

The absolute measurement error obtained by averaging Φ_1 and Φ_2 is between 2 and 5 nm. The rms error of the total phase for the modified method is between 1 and 2 nm, which corresponds to a decoding error of less than $\lambda/300$.

CONCLUSIONS

Thus, experimental validation of the modified method has shown that the proposed phase ambiguity algorithm makes it possible to eliminate gross errors caused by fringe profile distortions.

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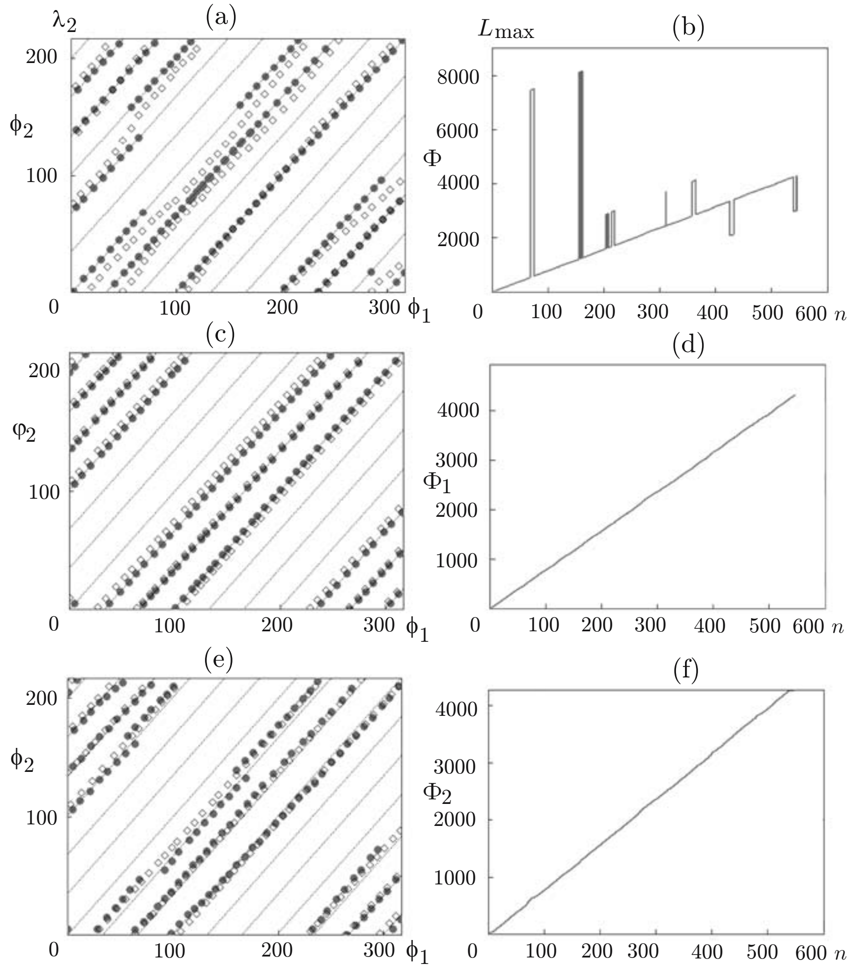

Fig. 5. Results obtained at Step 3: phase planes $((\phi_1, \phi_2)$ (a), (ϕ_1, φ_2) (c), (φ_1, ϕ_2) (e); total phase Φ (b), Φ_1 (d), and Φ_2 (f).

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