Robust Method Of Absolute Phase Mapping By Projection Of Series Sine Patterns With Different Periods

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Abstract— **A method is based on rules of integer arithmetic and allows determining the absolute phase in each point of the phase distributions set. The method does not require the comparison of the local phase values in adjacent areas of the phase distribution.**

Keywords-component; interference measuring system, data processing algorithms

I. Introduction

Reconstruction of an objects' surface is one of important technical tasks. Methods of structured illumination are widely used to resolve it [1-6]. One of most well-known methods is illumination of an object by a sine pattern. Surface of the object is determined by values of distortion of sine patterns, which are dependent on geometric parameters of the object. This method imitates an interference method of the surface reconstruction [7].

Intensity of a captured image can be represented as

$$I(x,y) = \alpha(x,y)\left(A + B\cos(2\pi f + \varphi(x,y)\right), \quad (1)$$

where (x, y) are coordinates of the image, $\alpha(x, y)$ – the reflectance variation or the albedo, A - average brightness, B - amplitude of the image, f - the frequency of the sine pattern, $\varphi(x,y)$ - the depth of the distortion of the sine pattern which is proportional to the object surface depth so-called *interference phase* [3].

The method allows to measure spatial coordinates of the surface depth with precision commensurate with period of the projected pattern. Taking into account designations in Fig. 1 surface depth can be represented as

$$h = \frac{BC\cdot(L/D)}{1+BC/D} \quad (2)$$

where $BC = \beta\left(\varphi_A - \varphi_B + 2\pi N\right) = \beta\Phi(x,y)$, β - the geometrical parameter, $\varphi_A - \varphi_B = \varphi$ - the phase difference at points A and B, φ - the local phase corresponding to the fractional part of the full phase $\Phi(x,y)$.

To calculate phase we use the phase-shifting method [7]

$$I_i(x,y) = \alpha(x,y)\left(A + B\cos(\varphi(x,y) + \delta\varphi_i\right) \quad (3)$$

A phase shift $\delta\varphi$ is obtained by means of spatial shift of projected pattern to a value which is proportional to the period of sine pattern *T*. The shift on a period equals to the phase shift $\delta\varphi = 2\pi$. Projecting of images series with shifts 0, $\frac{T}{3}$, $\frac{2T}{3}$ we obtain interferograms with phase shifts 0, $\delta\varphi_1 = \frac{2\pi}{3}$ и $\delta\varphi_1 = \frac{4\pi}{3}$, respectively. Then we get the following reconstruction formula:

$$\varphi = \arctan\frac{I_3 - I_2}{I_1 - I_2}. \quad (4)$$

Note that the phase of the interference pattern which is calculated by Eq. 4 is in range 0 ÷ 2π or –π ÷ +π. It is the phase ambiguity. The common solution of elimination of the problem is based on addition or subtraction of the values which are multiple of 2π [7-21]. However, there are some difficulties.

Process of distinguishing of the phase transitions requires elementwise phases' comparison in adjacent points of the phase field with a specified threshold. The threshold should be chosen such way that noises in the phase distribution do not cause to false phase transitions. False phase transitions lead to accumulation of errors on whole unwrapped area that does not allow to interpret of the phase field correctly. On the other hand, the threshold should be chosen to avoid of skipping true phase transitions. These requirements are contradictory, so choice of the threshold is rather difficult. To unwrap the phase we can use an algorithm of detection of the local phase transitions [8]. In Ref. 21 a method which allows to obtain the absolute phase value without phase unwrapping is proposed. However, the method is sensitive to noises. In this paper we propose a method of phase unwrapping which is stable to noises.

II. Proposed Methods and Solutions

As shown in Ref. 5 the absolute (unwrapped) phase values which are obtained by measured phases from interferograms with different periods can be represented by means of following equations system

$$\delta_1 \equiv \Lambda \bmod \lambda_1 .$$

$$\delta_2 \equiv \Lambda \bmod \lambda_2 .$$

Taking into account that interference measuring systems work with wrapped around 2π phases, measured values δ_1 and δ_2 should be normalized as $\delta_1 = (\lambda_1/2\pi)\varphi_1$ and $\delta_2 = (\lambda_2/2\pi)\varphi_2$, respectively.

Let us represent solution of the equation system as a trajectory of the point (A) on the complex plane with coordinates δ_1 and δ_2. If the absolute phase Λ changes, the point (A) will move on a certain trajectory in the plane (δ_1, δ_2) (fig. 1).

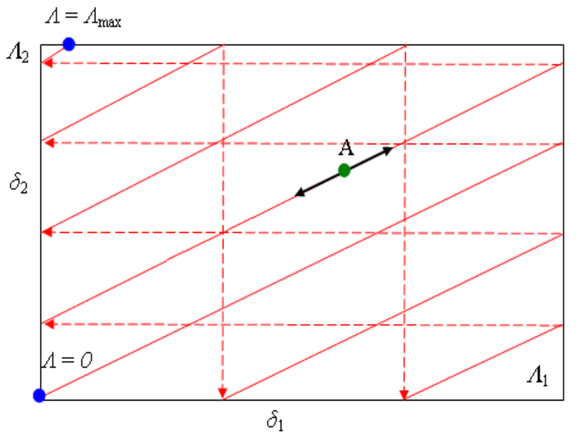

Figure 1. Trajectory of the point (A) on the complex plane.

In the case of phases changing on a value in range from 0 to 2π the point (A) forms a trajectory like a system of lines with breaks on the borders. Integrating travelled distance of the point (A) we can obtain the value of absolute phase.

We propose to unwrap phase following way:

1) Form a *N*x*N*-size table $T(\delta_1, \delta_2)$, where $N = \mathrm{int}\left(\frac{2\pi}{\Delta}\right)$, Δ - phase measuring error δ;

Values in cells of the table are calculated as

$$\begin{cases} T(\delta_1, \delta_2) = 1 \\ 0, otherwise \end{cases}$$

2) Detect phase transitions in lines on the borders;

3) Connect lines to each other and numerate them;

4) Calculate a value of the absolute phase by formula: $\Lambda = 2\pi n + \delta$.

Such sequence of operations allows to obtain a simple unwrapping algorithm, which does not have an effect of errors accumulation in case of false phase transitions. It is possible because of phase information at the point (x,y) is not used to determine line number.

We point next main features of this solution:

1) Phase transitions on each border of the table are formed in case of a value of the absolute phase is a number multiple of wavelength;

2) Number of phase transitions N in the table is limited and does not depend on the difficulty of interference pattern (e.g. number of local transitions in a phase image) and can be calculated as $N_i = \Lambda_{max}/\lambda_i$.

In the Fig. 2 a table which is obtained in a real experiment is given. Arrows show a sequence of connections of the lines to each other.

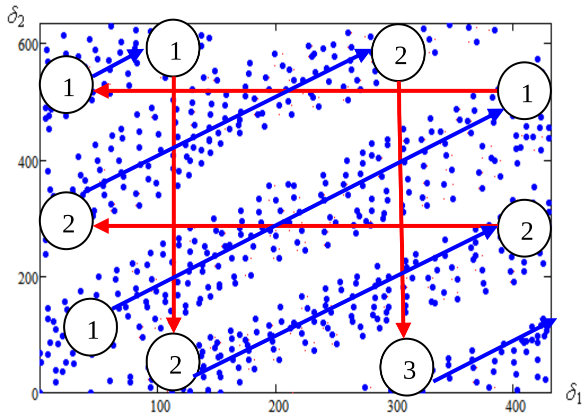

Figure 2. Numeration of lines.

Conclusions

We propose a simple algorithm of phase unwrapping. The algorithm requires having two phase distributions which are obtained by interferograms with different periods. The algorithm is stable to noises and errors accumulation.

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