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Inhaltsverzeichnis

Analysis Algorithm for Interference Patterns with Random Phase Shifts

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***Abstract* — analysis algorithms for interference patterns based on the phase shifts are widely used in creation of interference measuring systems. The phase-shifting method is based on obtaining interference patterns when the phase of a reference wave is changed to known values. An accuracy of existing analysis algorithms depends on a setting accuracy of inserted phase shifts values. However, it is difficult to determine an exact value of the phase shift in practice because of errors of devices performing a phase shift. A new analysis algorithm for interference patterns is considered in this paper. The algorithm uses three interference patterns with arbitrary phase shifts in order to calculate a phase.**

Keywords* — *interferometry; interference measuring system; interference patterns analysis

I. Introduction

A non-contact surface relief measurement based on an interference principle is a modern research area. Interference patterns captured during a measurement process contain a large amount of information that must be processed and decoded to obtain qualitative and quantitative assessments. This requires a presence of according computer systems and software to obtain, transform and process information. Software is used to decode interference patterns and present results in an appropriate form.

Interference measurement systems consist of an interferometer, a camera that is used to capture interference patterns, and a data processing system. An operating principle of an interferometer can be described as follows. An electromagnetic radiation beam from a laser is spatially divided in two coherent beams using a beam-splitter. The former is reflected from an object to be measured, the latter is reflected from a reference mirror. Each of the beams goes through different optical paths and returns to a screen, creating an interference pattern (interferogram). Figure 1 shows a simplified scheme of the Twyman-Green interferometer.

Then the captured interference patterns are used to restore a surface of an object to be measured, which is a task of a phase restoration for interference patterns.

From a mathematical point of view the task of a phase restoration is a determining of phase difference values of interfering wave fronts using a measured intensity of the captured interference patterns.

Analysis methods for interference patterns based on phase shifts are widely used for a construction of interference measurement systems [1]. A phase shift method is based on a capturing of several interference patterns when a reference wave phase is changing to known values.

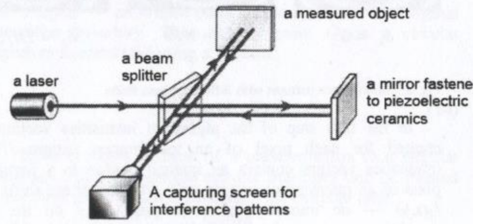

Fig. 1. The Twyman-Green interferometer scheme.

The interferogram's intensity at a point (x,y) with different phase shifts δ_i is

$$I_i(x,y) = I_0(x,y)\left[1 + V(x,y)\cos(\varphi(x,y)+\delta_i)\right], \quad (1)$$

where $I_0(x,y)$ — an average brightness, $V(x,y)$ — an interference pattern visibility, $\varphi(x,y)$ — a phase difference between interfering wavefronts, $i = 1,2,\ldots, m$, m — a phase shifts number.

There are formulas to determine the phase difference. If phase shifts are identical in the range $0 \ldots 2\pi$ the phase difference φ can be calculated as [2]

$$\varphi = arctg\left[\left(\sum_{i=1}^{n} I_i \sin\delta_i\right)\Big/\left(\sum_{i=1}^{n} I_i \cos\delta_i\right)\right] \quad (2)$$

The phase difference can be calculated using three arbitrary phase shifts:

$$\varphi = arctg \frac{(I_2 - I_3)\sin(\delta_1) + (I_3 - I_1)\sin(\delta_2) + (I_1 - I_2)\sin(\delta_3)}{(I_3 - I_2)\cos(\delta_1) + (I_1 - I_3)\cos(\delta_2) + (I_2 - I_1)\cos(\delta_3)} \quad (3)$$

An accuracy of existing analysis algorithms depends on a setting accuracy of introduced phase shifts. However, in practice it is difficult to determine an exact phase shifts values because of phase shift devices' errors.

This paper is dedicated to an analysis algorithm for interference patterns with three random phase shifts. A gist of the algorithm is to transform a trajectory of interference signals (intensities) with random phase shifts to a trajectory of signals whose phase shifts are exactly known.

II. Algorithm Description

Suppose there are three interference patterns with phase shifts δ_1, δ_2, δ_3 (figure 2).

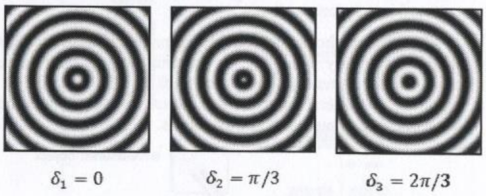

Fig. 2. Interference patterns with different phase shifts.

In the first step of the algorithm intensities vectors are created for each pixel of an interference pattern. These intensities vectors contain an intensity value in a particular pixel of an interference pattern with different phase shifts. Let $I_1(x,y)$ — an intensity value at pixel (x,y) on the first interference pattern, $I_2(x,y)$— an intensity value at pixel (x,y) on the second interference pattern, $I_3(x,y)$— an intensity value at pixel (x,y) on the third interference pattern. Then the intensities vector at pixel (x,y) on the interference pattern is $\mathbf{I}(x,y) = [I_1(x,y); I_2(x,y); I_3(x,y)]$.

Then orthogonal vectors are calculated for each intensities vector. Orthogonal vector can be calculated using a matrix equation

$$\mathbf{I}^{\perp} = \mathrm{M} \cdot \mathbf{I}. \quad (4)$$

A transformation matrix M must satisfy the following requirements:

$$|M| = 0, M \cdot [1 \ \ldots \ 1]^{\mathrm{T}} = 0 \quad (5)$$

Calculated orthogonal vectors will have the same dimension as the intensities vector, i.e. contain three components. Figure 3 shows a trajectory of orthogonal vectors in an intensities space. Coordinates of each point in figure 3 are elements of the corresponding orthogonal vector. A shape and a location of a points cloud depend on phase shifts in interference patterns.

In view of orthogonal vectors properties all points in the intensities space will be located in the same plane. This allows us to reduce a problem dimension. The resulting points cloud is rotated so that it would be parallel to a coordinate plane XY (figure 4). Then the points cloud is projected to a coordinate plane XY. After that points on the coordinate plane XY are approximated by an ellipse (a second-order curve) using an algorithm described in [4]. A second-order curve is determined by a following equation [3]:

$$a_{11}x^2 + a_{22}y^2 + 2a_{12}xy + 2a_{13}x + 2a_{23}y + a_{33} = 0 \quad (6)$$

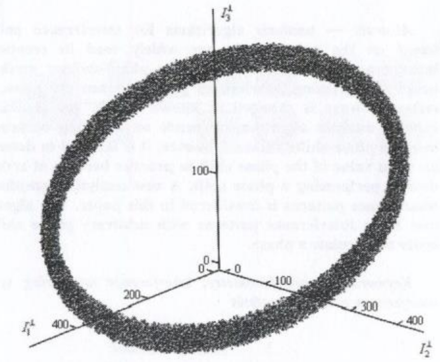

Fig. 3. A trajectory of orthogonal vectors in a space of intensities.

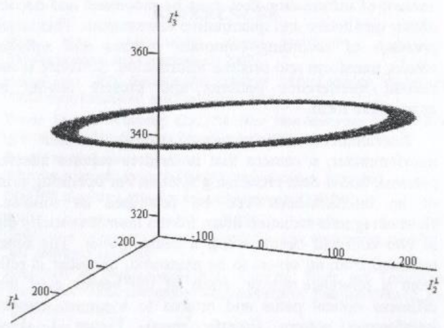

Fig. 4. An orthogonal vectors trajectory that is parallel to a coordinate plane.

The approximation algorithm uses an approximation criterion as in a least squares method, but the algorithm takes into account certain restrictions for coefficients of a second-order curve equation which represents exactly an ellipse. Figure 5 shows a result of an approximation.

The next step — inscribe an ellipse into a cylinder. For that it is necessary to determine a cylinder guideline. Suppose a — an ellipse's semi-major axis, b — an ellipse's semi-minor axis. Axes lengths of the ellipse can be determined from an equation of a second-order curve as follows. At first calculate characteristic equation roots of a second-order curve:

$$\begin{vmatrix} a_{11}-\lambda & a_{12} \\ a_{12} & a_{22}-\lambda \end{vmatrix} = 0 \tag{7}$$

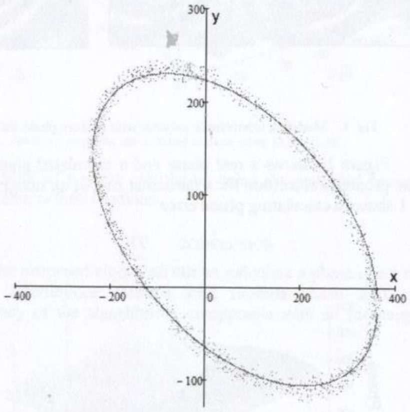

Fig. 5. A trajectory approximation using an ellipse.

Roots λ_1 and λ_2 of this equation are eigenvalues of a real symmetric matrix:

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{bmatrix} \tag{8}$$

Then axes lengths of an ellipse can be determined as follows. When $\lambda_1 > \lambda_2$:

$$a = \sqrt{(-1/\lambda_2)\cdot(A/D)}, b = \sqrt{(-1/\lambda_1)\cdot(A/D)} \tag{9}$$

where A and D are invariants of a second-order curve calculated using following formulas [3]:

$$D = \begin{vmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{vmatrix}, A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{vmatrix}. \tag{10}$$

When $\lambda_2 > \lambda_1$:

$$a = \sqrt{(-1/\lambda_1)\cdot(A/D)}, b = \sqrt{(-1/\lambda_2)\cdot(A/D)} \tag{11}$$

A guidline **L** of a cylinder into which an ellipse will be inscribed has a following form:

$$\mathbf{L} = \begin{bmatrix} \cos(\Omega) \\ \sin(\Omega) \\ b/a \end{bmatrix} \tag{12}$$

where Ω— an angle between a positive direction of the axis X and each of the two main lines of a second-order curve calculated using a formula [3]:

$$\Omega = \frac{1}{2} arctg \frac{2a_{12}}{a_{11}-a_{22}} \tag{13}$$

Then the intensities trajectory can be transformed to a circular trajectory using the guidline of a cylinder into which an ellipse is inscribed. Each point of the intensities trajectory is projected along the cylinder guidline on a plane that is parallel to a cylinder base. A transformation is carried out as follows. Let $P(x,y,z)$ — an ellipse point in the space for the original intensities trajectory. Then a new point G on a circular trajectory is calculated using a formula:

$$G = P - (\mathbf{L}\cdot P)\cdot \mathbf{L} \tag{14}$$

Figure 6 shows a result of a transformation to a circular trajectory. All points on the circular trajectory will be located in the same plane which is parallel to the cylinder base.

The next step — rotate the circular trajectory in such a manner that a normal direction of a plane in which points of the circular trajectory are located coincides with direction of a vector **E**=[1;1;1]. To perform this operation it is necessary to find a rotation matrix to rotate a plane normal of the circular trajectory to a vector **E**.

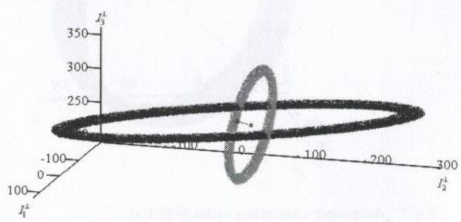

Fig. 6. A transform to a circular trajectory (a projection along a cylnder guidline)

This rotation matrix can be found as follows. Let **T** — vector that is rotated to a vector **P**. At first calculate an angle β between vectors **T** and **P** using following formula

$$\beta = \arccos\frac{\mathbf{T}\cdot\mathbf{P}}{|\mathbf{T}|\cdot|\mathbf{P}|}. \tag{15}$$

Calculate cross product of these vectors

$$\mathbf{V} = \mathbf{T}\times\mathbf{P}. \tag{16}$$

Then normalize the vector **V**:

$$R = \frac{\mathbf{V}}{|\mathbf{V}|} \tag{17}$$

Create a skew-symmetric matrix A:

$$A = \begin{bmatrix} 0 & -R_z & R_y \\ R_z & 0 & -R_x \\ -R_y & R_x & 0 \end{bmatrix} \tag{18}$$

Then a required rotation matrix S can be found using an exponential map (Rodrigues formula):

$$S = J + \sin(\beta)\cdot A + (1-\cos(\beta))\cdot A^2 \tag{19}$$

where J — an identity matrix of size 3×3.

Further each point of the circular trajectory is rotated using the rotation matrix S. Figure 7 shows a result of this transformation.

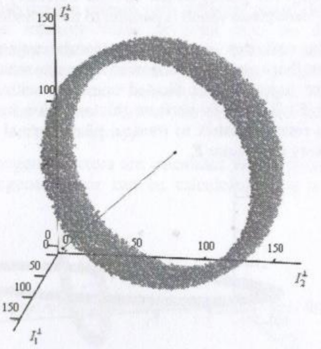

Fig. 7. A trajectory rotation to a vector **E**=[1;1;1].

After a transformation to a circular trajectory the phase difference of interfering wave fronts can be found using a well-known decoding formula with phase shifts values $\delta_1=0$, $\delta_2=2\pi/3$, $\delta_3=4\pi/3$.

III. MODELING EXPERIMENT

Interference patterns *300* × *300* pixels were simulated to test the proposed analysis algorithm. Figure 8 shows modeling interference patterns with random phase shifts δ_1, δ_2, δ_3. Figure 9 shows a phase distribution for entire field corresponding to model interference patterns.

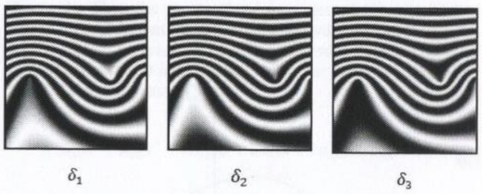

Fig. 8. Modeling interference patterns with random phase shifts.

Figure 10 shows a real phase and a calculated phase using the proposed algorithm for a particular row of an image. Figure 11 shows a calculating phase error.

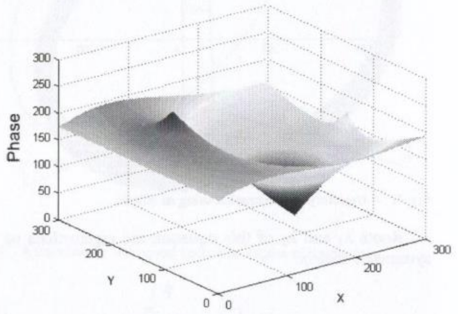

Fig. 9. A phase distribution for an entire field.

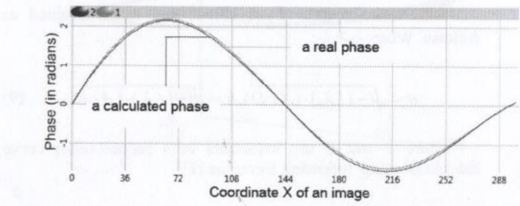

Fig. 10. A real phase and a calculated phase for a particular row.

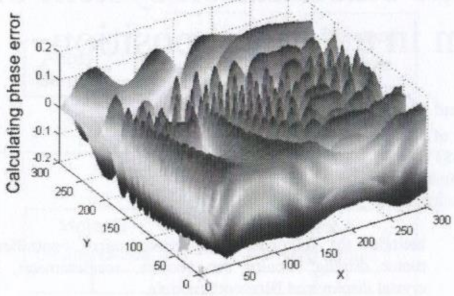

Fig. 11. An error of phase calculating at each point in the field.

RMSE of a phase calculating using the proposed decoding algorithm is *0.033* radians.

IV. CONCLUSION

The proposed algorithm allows calculate a phase using only three interference patterns with random phase shifts. An accuracy of the algorithm is comparable with an accuracy of conventional analysis algorithms which require exact values of phase shifts. Results of a model experiment demonstrate a high accuracy of the algorithm. Unknown random phase shifts can be introduced by vibrations of a measurement system during an experiment and can be used for an analysis of interference patterns.

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