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**OPTICAL INFORMATION TECHNOLOGIES**

# Quasiheterodyne Method of Interference Measurements

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**Abstract**—A method for measuring the phase difference between two interfering wavefronts on the basis of an analysis of the trajectories formed by the intensities of pairs of points in a series of interferograms with different phase shifts is proposed. This method does not require a priori knowledge of the actual values of phase shifts.

*Keywords*: interferometry, incremental phase shift.

**DOI**: ?

## INTRODUCTION

In the analysis of interference patterns by means of an incremental phase shift [1–5], inaccuracy in the value of introduced phase shifts causes an error of measurement of the phase difference between the object and reference wavefronts. Therefore, the analysis of measurement errors occurring due to the inaccuracy of phase shifts [6–11] are the subject of much attention at the present moment. There are two basic approaches to this problem. The first one requires calibrating the device for introducing phase shifts [11], which complicates the interferometric system. The second approach uses self-calibrating algorithms for measuring the phase incursion of interferograms, but the efficiency of computational algorithms is greatly reduced in this case [10–12].

The purpose of this paper is to create a method for measuring phase incursions, which does not require a priori information about the parameters of introduced phase shifts. The method consists in measuring the phase difference between the arbitrarily selected reference point and the spatial interferogram points (phase incursion). In Fig. 1, the point $A$ is a reference point; the phase incursion at the point $B$ is calculated relative to $A$. All the other values of phase incursion at different points of the interferogram are measured with respect to the same reference one. The choice of a reference point affects the constant shift of phase incursion at all points of the interferogram, which is not critical in the measurement of the phase difference of interfering wavefronts.

The proposed approach is based on the principles of the known heterodyning methods [13] (algorithmically implemented in this case) and was given the name of quasiheterodyne method.

## MATHEMATICAL DESCRIPTION OF THE METHOD

A brightness equation at each point of the interferogram $I(x, y)$ with different values of the introduced phase shifts can be represented as

$$I_i(x, y) = a(x, y) + b(x, y)\cos[\phi(x, y) + \delta_i], \tag{1}$$

where $a(x, y)$ is the average brightness; $b(x, y)$ is the amplitude of interference fringes; $\phi(x, y)$ is the phase difference of interfering wavefronts; $\delta_i$ is the $i$th phase shift; $i = 0, 1, \ldots, N$ ($N$ is the number of phase increments). It is supposed in Eq. (1) that the phase shift is the same for the entire interferogram field [3].

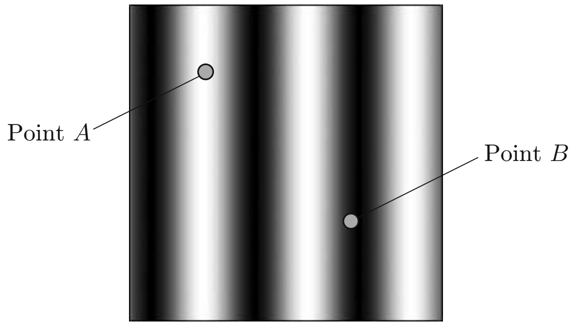


**Fig. 1.** Reference and arbitrary points in the interferogram, between which phase incursion is calculated.

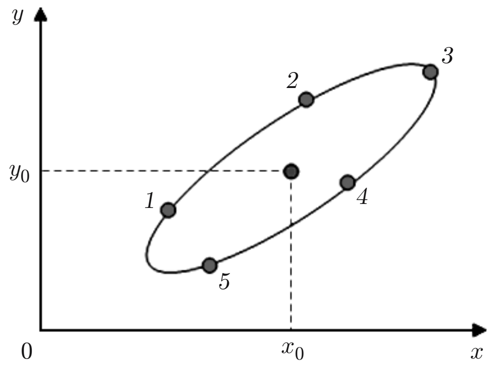


**Fig. 2.** Positions of intensity points on the plane for different ntroduced phase shifts.

If we take two points on the interferogram with the coordinates $(x_A, y_A)$ and $(x_B, y_B)$, then, after five phase shifts, we have a system of ten equations of the form (1) with ten unknowns [12, 13].

To simplify the presentation, we change the denotation of the brightness of points in system (1). Let the brightnesses $I_1(x_A, y_A), \ldots, I_5(x_A, y_A)$ and $I_1(x_B, y_B), \ldots, I_5(x_B, y_B)$ be represented as $x_1, \ldots, x_5$ and $y_1, \ldots, y_5$, respectively, and let the average brightness levels at the points $(x_A, y_A)$ and $(x_B, y_B)$ be denoted by $x_0$ and $y_0$. Given these notations, system (1) takes the following form:

$$x_i = x_0 + b_1 \cos(\phi_1 + \delta_i); \quad y_i = y_0 + b_2 \cos(\phi_2 + \delta_i), \quad i \in [0, 1, \ldots, 4]. \tag{2}$$

This can help us clearly identify the phase increment between two arbitrarily selected points of the interference pattern. Solution of system (2) is a rather difficult task as it is transcendent. Note, however, that this system can be represented by an ellipse equation [10]. This simplifies the solution.

For visualization, the coordinates of the points $(x, y)$ are displayed on a plane (Fig. 2). The first phase shift without limits of generality is considered to be zero ($\delta_0 = 0$). If the condition $\delta_0 < \delta_1 < \delta_2 < \ldots < \delta_N$ is satisfied, then the first point moves to the position of the second point, then to the third position, and so on; this is a translational movement of the point along the elliptical trajectory. Obviously, the point movement is chaotic with arbitrary values of phase shifts. In this case, analyzing the behavior of the trajectory and calculating its parameters are complicated. We offer the following method of solving this problem: ranking the points by the difference between the angles of vectors formed by the difference between the coordinates of each points and the coordinates of the assumed trajectory center, which can be obtained in the simplest case by averaging the coordinates of all points.

Note that any point on the plane that satisfies system (2) belongs to an ellipse. Then system (2) can be rewritten as

$$x_i = x_0 + b_1 \cos(\delta_i); \quad y_i = y_0 + b_2 \cos(\delta_i + \phi_2 - \phi_1), \tag{3}$$

where $\phi_2 - \phi_1$ is the phase increment at two different points of the interferogram.

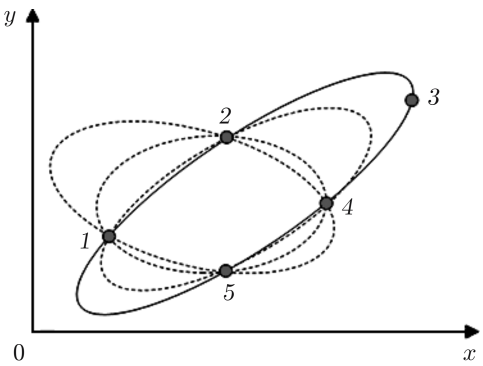


*Fig. 3.*

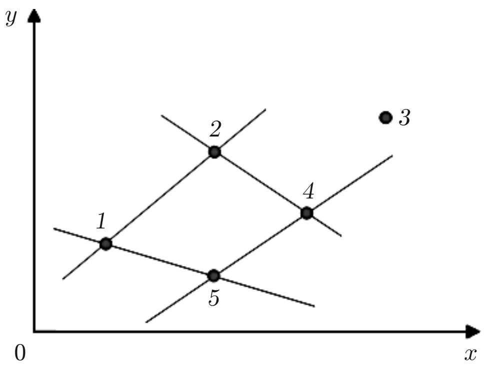


*Fig. 4.*

**Fig. 3.** Pencil of curves used for determining the coefficients of the ellipse.

**Fig. 4.** Order of connecting the points that form the pencil of curves.

The ellipse equation corresponding to system (3) has the form

$$\frac{(x_i - x_0)^2}{b_1^2} + \frac{(y_i - y_0)^2}{b_2^2} - 2\frac{(x_i - x_0)(y_i - y_0)}{b_1 b_2}\cos(\phi_2 - \phi_1) = \sin^2(\phi_2 - \phi_1). \quad (4)$$

However, direct determination of the coefficients of Eq. (4) is difficult because it is necessary to determine the unknown parameters of average brightnesses and amplitudes or their estimates [14]. On the other hand, the general ellipse equation can be represented as [15]

$$a_{11}x^2 + 2a_{12}xy + a_{22}y^2 + 2a_{13}x + 2a_{23}y + a_{33} = 0, \quad (5)$$

where $a_{ij}$ are the second order equation coefficients that describe the ellipse.

Taking into account the invariance of the leading coefficients of Eq. (5) to the plane-parallel transfer of the ellipse, we obtain the cosine of the phase difference $\phi_2 - \phi_1$ from its leading coefficients:

$$a_{11} = \frac{1}{b_1^2}; \quad a_{12} = \frac{1}{b_1 b_2}\cos(\phi_2 - \phi_1); \quad a_{22} = \frac{1}{b_2^2}. \quad (6)$$

Determination of the above-mentioned coefficients of Eq. (6) using the pencil of second-order curves is shown in Fig. 3.

The pencil of second-order curves can be formed from the combinations of the products of first-order curves (straight lines) [16]:

$$f_1(x, y)f_2(x, y) = \alpha f_3(x, y)f_4(x, y). \quad (7)$$

Here $f_i(x, y) = A_i x + B_i y + C_i$ is the equation of the $i$-th line, and $\alpha$ is chosen so that the curve taken from the pencil passes through the free point *3* in Fig. 4.

Then, for the points shown in Fig. 4, we have the following coefficients of straight-line equations:

$$A_1 = y_2 - y_1, \quad B_1 = x_1 - x_2, \quad C_1 = x_2 y_1 - x_1 y_2;$$
$$A_2 = y_5 - y_4, \quad B_2 = x_4 - x_5, \quad C_2 = x_5 y_4 - x_4 y_5;$$
$$A_3 = y_5 - y_1, \quad B_3 = x_1 - x_5, \quad C_3 = x_5 y_1 - x_1 y_5;$$
$$A_4 = y_4 - y_2, \quad B_4 = x_2 - x_4, \quad C_4 = x_4 y_2 - x_2 y_4;$$
$$\alpha = -(A_1 x_3 + B_1 y_3 + C_1)(A_2 x_3 + B_2 y_3 + C_2)/[(A_3 x_3 + B_3 y_3 + C_3)(A_4 x_3 + B_4 y_3 + C_4)].$$

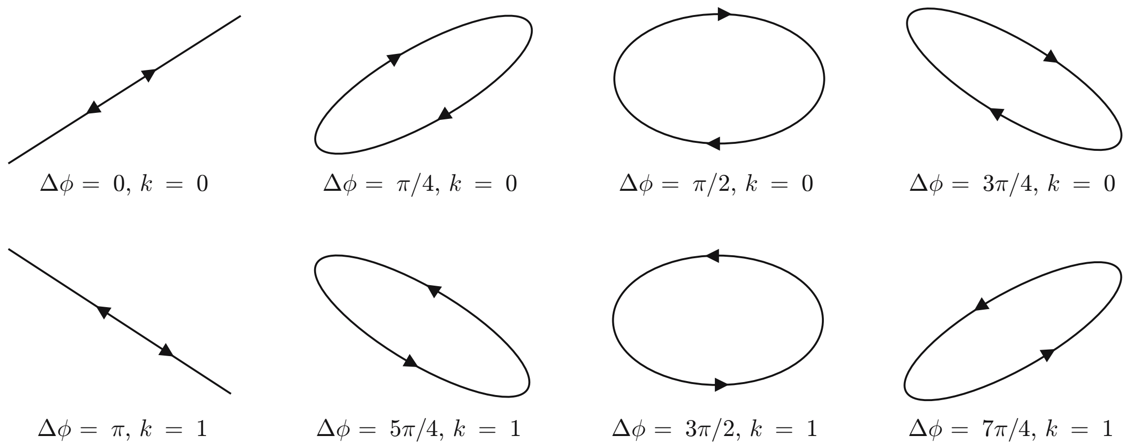


**Fig. 5.** Trajectories of the point on the plane, depending on the initial phase difference.

Substituting the coefficients obtained in Eq. (7), we write the required coefficients of the ellipse:

$$a_{11} = A_1A_2 + \alpha A_3A_4; \quad a_{12} = \tfrac{1}{2}[A_1B_2 + A_2B_1 + \alpha(A_3B_4 + A_4B_3)];$$
$$a_{22} = B_1B_2 + \alpha B_3B_4. \tag{8}$$

Further, the coefficients of the ellipse (7) are used to obtain the phase incursion $\Delta\phi$:

$$\cos(\Delta\phi) = -\frac{a_{12}}{\sqrt{|a_{11}|}\sqrt{|a_{22}|}}, \tag{9}$$

$$\Delta\phi = \phi_2 - \phi_1 = \arccos\Big(-\frac{a_{12}}{\sqrt{|a_{11}|}\sqrt{|a_{22}|}}\Big). \tag{10}$$

The arccos function is determined in the range from 0 to $\pi$, but it is possible to expand the measurement range of up to $2\pi$ by finding the value of the correction factor $k$:

$$\Delta\phi = \Delta\phi + k\pi. \tag{11}$$

This can be done by monitoring the orientation of the major axis of the ellipse and the direction of change in the trajectory of the point with respect to the zero phase shift for various values of the phase shifts (see Fig. 2). Figure 5 shows the trajectory of the point on a complex plane, which were obtained with continuous changes in the phase shift for different values of the initial phase difference $\Delta\phi$. The figure shows that the direction of movement of the point changes if the initial phase difference exceeds $\pi$. Thus, determining the direction of movement of the point, we expand the measurement range of phase difference up to $2\pi$. The direction of movement (clockwise or counterclockwise) can be determined by calculating the sign of the vector product of vectors: $V_i = \begin{bmatrix} x_i \\ y_i \\ 0 \end{bmatrix}$ and $V_{i+1} = \begin{bmatrix} x_{i+1} \\ y_{i+1} \\ 0 \end{bmatrix}$, then the sign of the direction of movement is given by

$$S = \mathrm{sign}\Big(\sum S_i/N\Big), \tag{12}$$

where $S_i = \mathrm{sign}(V_i \times V_{i+1})$.

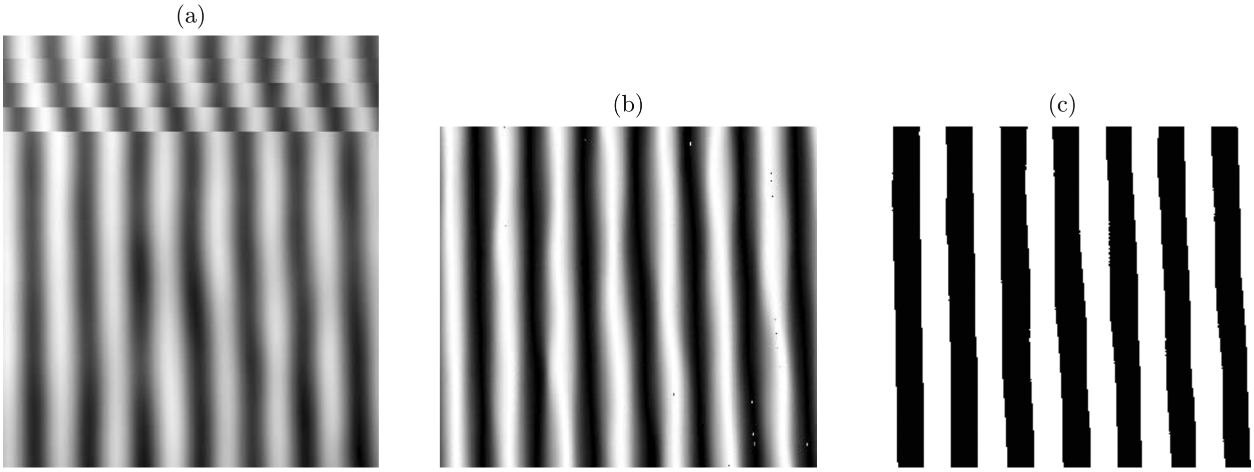


**Fig. 6.** Step of determining the direction of movement of points: (a) five interferograms with different phase shifts; (b) the cosine of phase difference calculated by the proposed algorithm; (c) the direction of movement of points (rotation of their radii-vectors) with introduced phase shifts (bright areas — the direction of rotation is clockwise, dark areas counterclockwise).

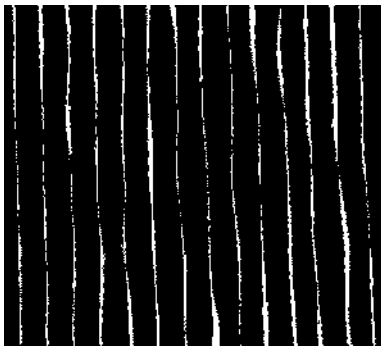

**Fig. 7.** Domains of bad points (bright areas).

## EXPERIMENTAL RESULTS

Experimental verification of the method was carried out by measuring the phase incursion of a series of five interferograms with different phase shifts (Fig. 6).

As the ellipse equation coefficients $a_{ij}$ are found with an error, there are bad points (Fig. 7) at which the cosine values defined by Eq. (10) (see Fig. 6b) can be greater than unity in modulus.

Figure 8 shows the measuring results for phase incursion in one of the cross-sections of interferograms with predetermined phase shifts $\delta \in [0, \pi/3, 2\pi/3, \pi, 4\pi/3]$ (see Fig. 6a) by the proposed (quasiheterodyne) and known [1] algorithm. The figure shows that measuring the phase incursion by the known algorithm [1] causes corrugation in the phase profile of interference fringes (curve *1*), which is due to the inaccuracy of phase shifts. The phase profile of the same fringes in the measurement of phase incursion by the proposed algorithm (curve *2*) is subjected to this effect to a lesser extent. Further suppression of this effect can be achieved by changing the reference point and repeating the calculations.

Figure 9 shows a comparison of the measurement results for phase incursion with a single reference point (curve *1*) and averaging over lines when the reference point runs through all the values in the line and the results are averaged (line *2*).

The resulting maximum error in determining the phase incursion $\Delta\phi$ by the proposed algorithm was 0.006 rad with averaging over 256 lines.

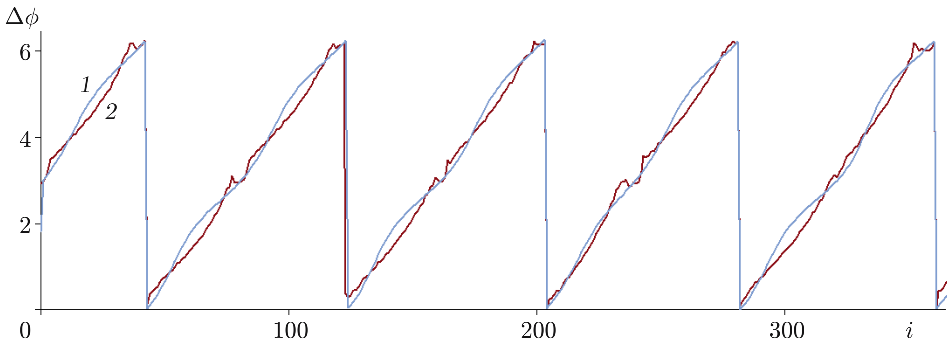


**Fig. 8.** Comparison of the measurement results for phase incursion by the algorithm [1] (curve *1*) and by the quasiheterodyne method (curve *2*).

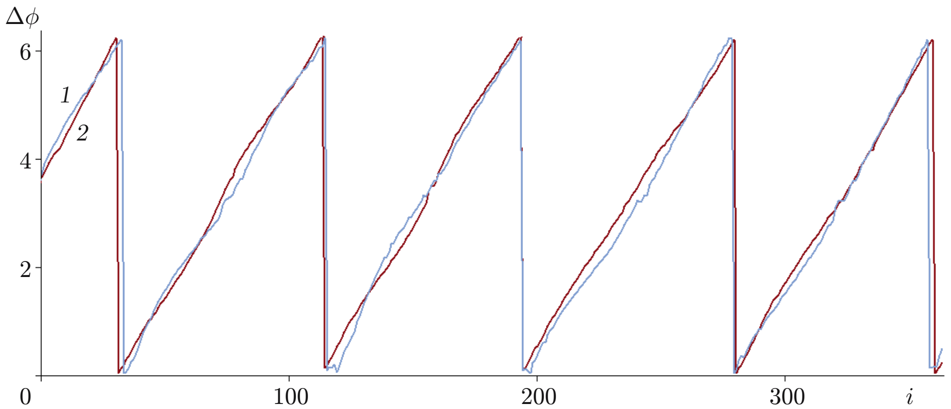


**Fig. 9.** Averaging the measurement results for phase incursion in the line.

## CONCLUSION

This paper proposes a method for determining the phase incursion in two arbitrary points of the interferogram. If a point is fixed, it is possible to find the distribution of the field of phase differences of interfering wavefronts over the entire interferogram field. This method allows using measuring systems with a priori unknown or arbitrarily inserted phase shifts.

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