A New Approach to Improving the Quality of Measurements in Multi-Wave Interference Systems

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***Abstract*—This paper proposes a method of expanding the range and improving the accuracy of interferometric measurements using three wavelengths. The method is based on the methods of modular mathematics for phase unwrapping. Unlike the known methods of unwrapping the phase accuracy of the proposed method is almost independent of the value of the dynamic range.**

***Keywords*—mechatronics; robotics; optical interference system; measuring range; laser**

I. Introduction

Single-frequency optical measurement techniques are widely used in various fields of modern science and technology [1]. Consider the possibility of interference methods to measure distances. These methods are well established for the measurement of profiles of the same scale. However, when the magnitude of the measured distances have a different scale, existing single-frequency interference measurement methods fundamentally do not reveal the differences of heights greater than the wavelength of the laser radiation [2], which significantly limits the range of applications simple smooth surfaces or surfaces having steps at a small size. In addition, such measuring systems have a relatively small dynamic range of measured distances. Therefore, to measure the geometrical parameters of micro- and nano-objects used methods of digital holographic microscopy and digital interferometry [3], and to measure geometrical parameters of macro-objects are used methods of structured illumination [4]. Note that the structure of modern optical measuring systems is mainly determined by the method of analyzing the optical information [5]. Therefore, the article primarily discusses new approach to the analysis of optical measurement data, which can address this limitation by expanding the dynamic range of optical measuring systems. The proposed approach is based on the methods of modular arithmetic and allows you to build a new class of high-precision optical measuring systems with extended measuring range. Note that the methods proposed for the phase measurement can be used in other frequency bands, for example, acoustic, microwave ranges, etc. Thus, considered in the paper issues are not limited to optical measurement techniques, and are relevant and promising for a wide range of technical applications.

II. Analysis of the Problem

Consider the equation for the interference of two wave fronts - subject (reflected from the surface) and reference (reflected from the reference surface, for example. the mirrors of the interferometer)

$$I = A\left[1 + V\cos\left(2\pi \cdot \Delta L/\lambda\right)\right] = A\left[1 + V\cos\left(\varphi\right)\right], \quad (1)$$

where I is the intensity, A is amplitude, V is contrast, λ is wavelength and ΔL is optical path difference and phase interference fringes respectively.

The expression (1) can be rewritten as

$$\Delta L = \left(\varphi/2\pi + k\right)\lambda . \quad (2)$$

Rate limit value ΔL which can be measured by a single-frequency method

$$\Delta L_{\max} = k_{\max}\lambda . \quad (3)$$

The maximum number of interference fringes, which can be distinguished on the interferogram is determined first, digital recording device, and secondly, the noise level of the brightness field of the interferogram. Most modern digital camera format for input interferogram 4K has a resolution of 4096x2160 pixels. For confident distinguish between interference fringes, it is necessary that one lane had at least four points. Then the characteristic number of interference fringes which can be registered on the interferogram will be $k_{\max} \le 500$. When using a semiconductor laser, for example, with a wavelength of 750 nm, get $\Delta L_{\max} \le 4$. See on the other hand the number of generated as a result of measurement of the interference fringes depends on the spatial coherence length of the laser depends on the spectral width of the laser illumination $\Delta\lambda$

$$L_c = \lambda^2/\Delta\lambda . \quad (4)$$

For stable semiconductor laser has a characteristic spectral width value in a few hundredths of a nanometer. Based on these assumptions, the estimate of the coherence length is the size from a few centimeters to several meters. Hence we can conclude that the dynamic range of a modern single-frequency interference systems for one, two orders of magnitude below the required. Another disadvantage of single-frequency interference systems is the inability correct measurement of the stepped surface profiles, when the height difference is greater than the wavelength. To obtain correct measurement results is necessary to satisfy the ratio $\Delta L_{\max} < \lambda$. This result is also a consequence of the small dynamic range of single-frequency interference measurement methods. This disadvantage can be eliminated if you use multi-wavelength methods of deployment phase [6]. For example, while the surface is illuminated by two lasers with different wavelengths λ_1 and λ_2 obtained equivalent wavelength

$$\lambda_{eq} = \lambda_1 \lambda_2 / |\lambda_1 - \lambda_2| . \quad (5)$$

Choosing the appropriate wavelengths it is possible to increase the dynamic range. Let wavelengths accept the values of $\lambda_1 = 633$ nm and $\lambda_2 = 452$ nm, which corresponds to the wavelengths of red and green lasers. Then, in accordance with formulas (4) and (5) we get $\Delta L_{\max} = 633 \cdot 452 / |633 - 452| = 1580.75$ nm. The gain in increasing the dynamic range in comparison with the maximum emission wavelength is only 2.5 times. In addition, in the practical measurement by this method is difficult to get a reasonably accurate definition because with increasing wavelength is the amplification of the measurement error is proportional to the value of the gain in the increased range. Measurement error δL will be proportional to the quantization error δI of the digital reception intensity I

$$\delta L = \Delta L_{\max} \cdot \delta I , \quad (6)$$

where $\delta I = 2^{-n}$ is the quantization error of a digital input device, n is the number of bits of this device. For example, when the 8-bit input device the error is 0.39% and the maximum error of measurement in accordance with the formula (6) can reach values of a few nanometers, for a two-wavelength method, the error will amount to several tens of nanometers, respectively. In work [6] to reduce the error of the proposed three-wavelength method, in which the equivalent wavelength is formed by the differences between the original wave calculated by the formula (5)

$$\lambda_{13-23} = \lambda_{13} \lambda_{23} / |\lambda_{13} - \lambda_{23}| . \quad (7)$$

The author [3] was able to reduce the measurement error to a level comparable with the accuracy of single-frequency method, however, the dynamic range turned out to be the same as in the two-wavelength method. Obtained in this section the evaluation of existing methods show that the developments of new methods for extending the dynamic range of the measurements are very important.

III. Proposed Approach

For the analysis of interference patterns is widely used definition of the phase difference φ values of the recorded intensities I_i of the method of phase shifts. When different phase shifts δ_i of the intensity of the reflected light from the object can be represented in the form

$$I_i = A\left(1 + V\cos(\varphi + \delta_i)\right) \quad (8)$$

In papers [7-9] generalized the algorithm of decoding, which at known values to determine the value of the phase distribution. In paper [10] the production method of solutions of the system of equations (8) with unknown values δ_i . The value will change from 0 to 2π. As a result we have the results of indirect measurements of parameters of the object depending on the price of the strip m, which is a parameter determined by the scheme of the interferometer

$$b_i(x, y) = \frac{\varphi(x, y)}{2\pi} m_i . \quad (9)$$

If you change the value of the price band and repeat the measurement (1), we obtain a series of values (b_1,b_2 , ... , b_k). Each value of b varies in the range from 0 to m_i-1. The absolute value of L is the solution of the system of congruences

$$\begin{cases} L \equiv b_1 \bmod (m_1) \\ \quad \cdots \\ L \equiv b_n \bmod (m_n) \end{cases} . \quad (10)$$

The maximum unambiguous range determination of absolute values is defined by the greatest coprime factors in the values of the periods. Thus, the task of extending the measurement range is reduced to the problem of solving the system of congruencies. However, the solution such system is unstable. Even a small error in the measurement of the initial values of b_i leads to significant errors in determining the full value of the measured value.

Theorem

Though

$$M_s N_s \equiv 1 \pmod{m_s} \quad (11)$$

and let

$$X_0 = M_1 N_1 b_1 + M_2 N_2 b_2 + \ldots + M_k N_k b_k \,. \qquad (12)$$

Then the set of values of X that satisfy the system of congruences (12), is determined by comparing

$$X \equiv X_0 (mod\ m_1 m_2 \ldots m_k) \qquad (13)$$

divided into modules $m_1 m_2 \ldots m_k$, remembering residues $b_1, b_2, \ldots, b_k$. Reverse transition from modular representations to the positional system is more complicated. You can use the expression (12). In the range defined by the product of coprime modules mod $m_1 m_2 \ldots m_k$ will be the only solution. Let the number of M_s and N_s determined from the conditions

$$m_1 m_2 \ldots m_k = M_s m_s \qquad (14)$$

The coefficients M_i can be found from the equality (14), Ni from the equality (12) by going through 1, 2, 3, and so on until the first of satisfying values, or by using a generalized Euclidean algorithm [2]. Time to find these factors are not important because this step is performed only once and usually we work with the same set of modules. Consider the case for three coprime modules. Let for example: $m_1 = 11$, $m_2 = 13$, $m_3 = 17$. In this case: $M_1 = 221$, $M_2^3 = 187$, $M_3^3 = 143$, $N_1 = 1$, $N_2^3 = 8$ N_{23}= 8, $N_3^3 = 5$.

$$m_1 m_2 \ldots m_k = M_1 N_1 b_1 + M_2 N_2 b_2 + M_3 N_3 b_3 \ \text{mod}\left(m_1\ m_2\ m_3\right) \quad (15a)$$

$$X = 22 \cdot 1 b_1 + 187 \cdot 8 b_2 1 b_1 + 143 \cdot 5 b_3 \ \left(\text{mod}\ 11\ 13\ 17\right) \quad (15b)$$

or

$$X = 221 b_1 + 1496 b_2 + 715 b_3 \ \left(\text{mod}\ 2431\right) \,. \quad (15c)$$

The decision table will look as follows (see Figure 1).

Unfortunately, the values consistently increase along the main diagonals. Two-dimensional slices along x and y do not allow you to analyze data in the presence of noise. However, this table can be rolled into a three-dimensional torus.

To do this, imagine solutions for X in the two-dimensional table, in which the x-coordinate is set aside values of turns. Imagine the number in the modular representation for two modules $m_1, m_2 = M_s m_2^3$.

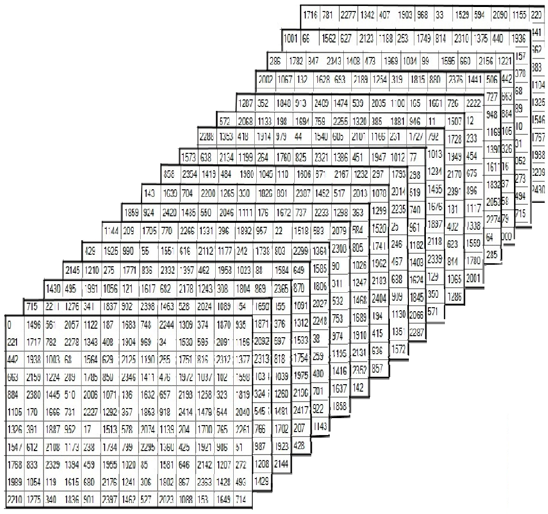

Fig.1 A decision table with modules $m_1 = 11$, $m_2 = 13$, $m_3 = 17$

$$X = M_1 N_1 b_1 + \left(M_2^3 N_2^3 + M_3^3 N_3^3\right) b_2^3 \qquad (16a)$$

Note that $M_2^3 N_2^3 + M_3^3 N_3^3 = M_2^2 N_2^2$, so

$$X = M_1 N_1 b_1 + M_2^2 N_2^2 b_2^3 \qquad (16b)$$

and then

$$X = 221 b_1 + 2211 b_2^3 \ \left(\text{mod}\ 2431\right) \,. \qquad (16c)$$

Note that in this case we will also work in the system of residual classes modulo $m_1 m_2 m_3 = 2431$. The values of $m_1 m_2 \ldots m_k$ ranged from 0 to 11, b_2^3 varies from 0 to 220.

The initial part of the decision tables (16b) is shown in Figure 2.

		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	b2	0	1	2	3	4	5	6	7	8	9	10	11	12	0	1	2	3	4	5	6	7
b1	b3	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	0	1	2	3
0		0	2211	1991	1771	1551	1331	1111	891	671	451	231	11	2222	2002	1782	1562	1342	1122	902	682	462
1		221	1	2212	1992	1772	1552	1332	1112	892	672	452	232	12	2223	2003	1783	1563	1343	1123	903	683
2		442	222	2	2213	1993	1773	1553	1333	1113	893	673	453	233	13	2224	2004	1784	1564	1344	1124	904
3		663	443	223	3	2214	1994	1774	1554	1334	1114	894	674	454	234	14	2225	2005	1785	1565	1345	1125
4		884	664	444	224	4	2215	1995	1775	1555	1335	1115	895	675	455	235	15	2226	2006	1786	1566	1346
5		1105	885	665	445	225	5	2216	1996	1776	1556	1336	1116	896	676	456	236	16	2227	2007	1787	1567
6		1326	1106	886	666	446	226	6	2217	1997	1777	1557	1337	1117	897	677	457	237	17	2228	2008	1788
7		1547	1327	1107	887	667	447	227	7	2218	1998	1778	1558	1338	1118	898	678	458	238	18	2229	2009
8		1768	1548	1328	1108	888	668	448	228	8	2219	1999	1779	1559	1339	1119	899	679	459	239	19	2230
9		1989	1769	1549	1329	1109	889	669	449	229	9	2220	2000	1780	1560	1340	1120	900	680	460	240	20
10		2210	1990	1770	1550	1330	1110	890	670	450	230	10	2221	2001	1781	1561	1341	1121	901	681	461	241

Fig.2. The initial part of the decision tables (15). (b_2^3 vary from 0 to 220).

In this table the first line specifies the number of turns, which will be determined as

$$n(i) = N_2^2 b_1 \mod(m_2\ m_3), \qquad (17)$$

where $i = b_2^3$.

The solution of the expression (17) for the two-dimensional case

$$X = M_2^2 \left[N_2^2 \left(b_2^3 - b_1 \right) \mod(m_2\ m_3) \right] \mod(m_2\ m_3) + b_1 \quad (18a)$$

or

$$X = \left\{ N_2^2 \left[\left(b_2^3 - b_1 \right) \mod(m_2\ m_3) \right] \mod(m_2\ m_3) \right\} m_2 m_3 + b_1. \quad (18b)$$

Now you need to determine the dependence b_2^3 from b_2 and b_2. After analyzing the decision table in Fig. 1 and Fig.-2 it can be seen that for the same solutions, these values will be distributed as shown in the second and third line of the table in Fig. 3.

		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
0		0	52	104	156	208	39	91	143	195	26	78	130	182	13	65	117	169
1		170	1	53	105	157	209	40	92	144	196	27	79	131	183	14	66	118
2		119	171	2	54	106	158	210	41	93	145	197	28	80	132	184	15	67
3		68	120	172	3	55	107	159	211	42	94	146	198	29	81	133	185	16
4		17	69	121	173	4	56	108	160	212	43	95	147	199	30	82	134	186
5		187	18	70	122	174	5	57	109	161	213	44	96	148	200	31	83	135
6		136	188	19	71	123	175	6	58	110	162	214	45	97	149	201	32	84
7		85	137	189	20	72	124	176	7	59	111	163	215	46	98	150	202	33
8		34	86	138	190	21	73	125	177	8	60	112	164	216	47	99	151	203
9		204	35	87	139	191	22	74	126	178	9	61	113	165	217	48	100	152
10		153	205	36	88	140	192	23	75	127	179	10	62	114	166	218	49	101
11		102	154	206	37	89	141	193	24	76	128	180	11	63	115	167	219	50
12		51	103	155	207	38	90	142	194	25	77	129	181	12	64	116	168	220

Fig. 3.Definition b_2^3 on b_2 (0 to 12) and b_3 (0 to 16).

In this case

$$b_2^3 = M_1^{23} N_1^{23} b_2 + M_2^{23} M_1^{23} b_3 \mod(m_2\ m_3). \qquad (19)$$

Here $M_1^{23} = 17$, $M_2^{23} = 13$, $N_1^{23} = 10$, $N_2^{23} = 4$.

$$b_2^3 = M_1^{23} \left\{ \left[N_1^{23} \left(b_3 - b_2 \right) \mod(m_2) \right] \right\} \mod(m_2) + b_1 \quad (20a)$$

or

$$b_2^3 = \left\{ N_1^{23} \left[\left(b_3 - b_2 \right) \mod(m_2) \right] \mod(m_2) \right\} \cdot m_2 + b_2. (20b)$$

Thus, the solution for three modules can be written as

$$X = \left\{ N_2^2 \left[\left(b_2^3 - b_1 \right) \mod(m_2 m_3) \right] \mod(m_2 m_3) \right\} \cdot m_2 m_3 + b_1. (21a)$$

$$X \left\{ \left(N_2^2 i \right) \mod(m_2 m_3) \right\} \cdot m_2 m_3 + b_1, \qquad (21b)$$

$$i = b_2^3 - b_1 \text{ if } b_2^3 - b_1 \geq 0 \quad i = b_2^3 - b_1 m_2 m_3 \text{ otherwise,}$$

$$X = \left\{ N_2^2 \left[\left(b_2^3 - b_1 \right) \mod(m_2 m_3) \right] \mod(m_2 m_3) \right\} \cdot m_2 m_3 + b_1. (22a)$$

Where

$$X \left\{ \left(N_2^2 i \right) \mod(m_2 m_3) \right\} \cdot m_2 m_3 + b_1. \qquad (22b)$$

$$i = b_3 - b_2 \text{ if } b_3 - b_2 \geq 0 \quad i = b_3 - b_2 + m_2 \text{ otherwise.}$$

The coefficients N_2^2 and N_2^{23} are calculated as follows

$$m_1 N_2^2 = 1 \mod(m_2 m_3). \qquad (23a)$$

$$m_2 N_2^{23} = 1 \mod(m_3). \qquad (23b)$$

Note that in the decision table (Fig. 2) the values consistently increase along the main diagonals. Therefore, when there are errors in the measurements of initial values ($b_1, b_2\ b_3$) it is always possible to limit the range (number of turns) to highlight the rude surroundings of failures. Erroneous results that fall within the neighborhood of gross failures that can be adjusted to the nearest diagonals. Resolving these errors can lead to a significant increase in accuracy. Thus, the use of modular arithmetic allows not only to increase the measurement range and to decrease the error. A similar calculation scheme can also be generalized for the multidimensional case for an arbitrary number of wavelengths.

IV. Conclusion

When expanding a dynamic range based on modular arithmetic at the same wavelengths $\Delta L_{\max} = \lambda_1 \lambda_2$ is 34.086µm, which is 21 times more range for two-wavelength method and 53 times more than single-frequency method, respectively. It should be noted that in our case, with the increase of the range error increases (not comparable with the accuracy of single-frequency method. If the same error ranges two-wavelength method by the Eq. (6) was already 131nm or 21%. Therefore the proposed method allows more than 50 times improving the accuracy of measurements in comparison with wavelength method. In the proposed method, the theoretical limit of the dynamic range is expressed as $\Delta L_{\max} \le \lambda_1 \cdot \ldots \cdot \lambda_2 \cdot \lambda_N$. For example, when using three semiconductor lasers emits radiation with wavelengths of 405nm, 650nm and 780nm, respectively, the dynamic range will be equal to 0.25m already, and the measurement accuracy does not depend on the value of the dynamic range.

References

[1] D. Malacara, c.4 in Optical Shop Testing 3nd ed. (D. Malacara,ed. Wiley, New York, 2007).J. Clerk Maxwell, A Treatise on Electricity and Magnetism, 3rd ed., vol. 2. Oxford: Clarendon, 1892, pp.68-73.

[2] V.I. Guzhov, S.P. Ilinykh, D.S. Khaidukov, R.A. Kuznetsov, "Estimation of validity of optical measurements," // 2012 11th International Conference on Actual Problems of Electronic Instrument Engineering, APEIE 2012 – Proceedings 6628978, pp. 146-149.

[3] T. Kreis, Handbook of holographic interferometry optical and digital methods. Wiley : VCH, 2005. 542 p.

[4] V. Guzhov, S. Ilinykh, A. Vagizov, R. Kuznetsov, "New principle of the shaping the nonline illumination in optical measuring systems," // Proceedings of the 6th International Forum on Strategic Technology, IFOST 2011.- 2, 6021110, pp. 652-654.

[5] V.I. Guzhov, S.P. Ilinykh, D.S. Khaidukov, R.A. Kuznetsov, "Method of an assessment of reliability of high-precision measurements," // 2012 11th International Conference on Actual Problems of Electronic Instrument Engineering, APEIE 2012 – Proceedings 6629152, pp. 105-106.

[6] Nilanthi Warnasooriya and Myung K. Kim (2010). Quantitative Phase Imaging Using Multi-Wavelength Optical Phase Unwrapping, Advances in Lasers and Electro Optics, Nelson Costa and Adolfo Cartaxo (Ed.), ISBN: 978-953-307-088-9, InTech, Available from: http://www.intechopen.com/books/advances-in-lasers-andelectro-optics/quantitative-phase-imaging-using-multi-wavelength-optical-phase-unwrapping.

[7] V. Guzhov, S. Ilinykh., R. Kuznetsov, D. Haydukov, "Generic algorithm of phase reconstruction in phase-shifting interferometry," // Optical Engineering, – 2013.-Vol.52(3) – pp. 030501-1 – 030501-2.

[8] V. Guzhov, S. Ilinykh, R. Kuznetsov, D. Haydukov, "Eliminating phase-shift errors in interferometry," // Optoelectronics, Instrumentation and Data Processing.-2011.- Volume 47, Issue 1, pp 76-80.

[9] V.I. Guzhov, S.P. Il'inykh, R.A. Kuznetsov, A.R. Vagizov, "Solution of the problem of phase ambiguity by integer interferometry," // Optoelectronics, Instrumentation and Data Processing.- March 2013, Volume 49, Issue 2, pp 179-184.

[10] V.I. Guzhov, S.P. Ilinykh, , D.S. Haydukov, R.A. Kuznetzov, "Decoding algorithm for interference patterns in phase shifting interferometry without a priori shift knowledge," // Proceedings - 2012 7th International Forum on Strategic Technology, IFOST 2012.- 6357647 pp. 674-676.

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