Decoding algorithm for interference patterns in phase shifting interferometry without a priori shift knowledge

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***Abstract*— Interferogram's decoding methods based on the step-by-step shift are widely used in creation of interference measuring systems. The phase-shifting method is based on obtaining several interferograms when the phase of reference wave is changed to known values. Accuracy of existing decoding algorithms depends on a setting of values of inserted phase shifts. However, it is difficult to determine an exact value of the phase shift in practice because of errors of devices performing a phase shift. In this paper an algorithm based on the step phase-shifting without a priori knowledge of phase shift is considered. This algorithm uses three interferograms obtained with arbitrary phase shifts.**

Keywords-interferometry; interference measuring system; decoding algorithm

I. Introduction

Lately interferogram's obtaining and decoding methods based on the step-by-step shift are widely used in a construction of interference measurement systems (phase-sampling, phase-shifting interferometry) [1]. The cause is a simplicity of the phase shift setting, simple algorithms and high decoding accuracy. Therefore existing schemes of interferometer are easy in modification.

The step phase-shifting method is based on a capturing of several interferograms while the phase of reference wave is changing to known values. The phase shift between interfering beams can be implemented in various ways. The simplest way is a mirror movement using piezoelectric ceramics.

Figure 1 shows simplified scheme of interferometer that realizes this approach. A light beam from a laser falls on a beam splitter and then it is divided into the reference and the object beams. There is a mirror fastened to piezoelectric ceramics to make phase shifts in a reference arm. An object arm configuration depends on a measuring problem and an shape of an object's surface. The interferogram capturing screen consists of an array of detectors to capture an intensity at each point. After each phase shift the intensities array is entered to a computer.

The interferogram's intensity at a point (x, y) with different phase shifts δ_i is [2]

$$I_i(x,y) = I_0(x,y)[1 + V(x,y)\cos(\varphi(x,y) + \delta_i)],$$

where $I_0(x,y)$ – an average brightness, $V(x,y)$ - an interference pattern visibility, $\varphi(x,y)$ – phase difference between interfering wavefronts, $i = 1,2,\dots,m, m$ – phase shifts number.

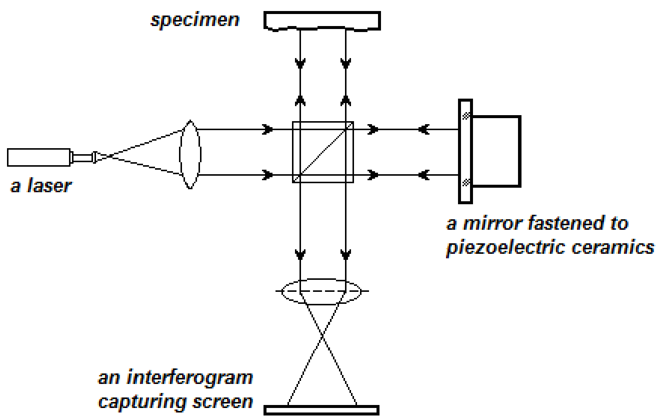


Figure 1. The Twyman-Green interferometer scheme with the moving of the mirror fastened to piezoelectric ceramics.

An interferogram's decoding task is to determine phase difference values by measured intensities of the interferogram. In mathematics this is non-linear inverse problem that is solved in conditions of a prior uncertainty $I_0(x,y)$ and $V(x,y)$.

There are decoding formulas to determine the phase difference [2]. If phase shifts are identical in the range $0 \dots 2\pi$ the phase difference φ can be calculated as

$$arctg\frac{\sum I_i \sin(\delta_i)}{\sum I_i \cos(\delta_i)}$$

When three arbitrary phase shifts the phase difference can be calculated as

$$arctg\frac{(I_1 - I_3)\sin(\delta_1) + (I_3 - I_1)\sin(\delta_2) + (I_1 - I_2)\sin(\delta_3)}{(I_3 - I_2)\cos(\delta_1) + (I_1 - I_3)\cos(\delta_2) + (I_2 - I_2)\cos(\delta_3)}$$

There are many algorithms with a large number of phase shifts. A few years ago the limiting factors in the use of such algorithms were: memory capacity to store intermediate frames and a required processing time. An increased computing power has allowed use algorithms with a large number of shifts.

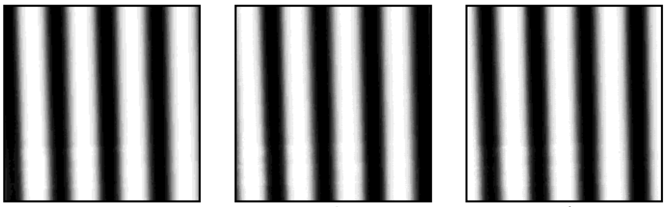
The phase measurement error depends on a setting accuracy of entered phase shift values, however in practice these values are difficult to determine because of errors of phase-shifting devices. Errors can be caused by, for example, hysteresis, piezoelectric vibration or device's vibration during an experiment.

This paper discusses the interferogram decoding algorithm based on step-by-step shift method which does not require a priori knowledge of the phase shifts.

II. ALGORITHM DESCRIPTION

The method essence is reducing the trajectory of interference signals with arbitrary phase shifts to the trajectory of interference signals whose phase shifts are exactly known.

Figure 2 shows interference patterns obtained at three different phase shifts δ $(0, {}^{2\pi}/_{3}, {}^{4\pi}/_{3})$. In these interference patterns values of intensity are in the range of 0 ... 255.



$\delta = 0$ $\delta = {}^{2\pi}/_{3}$ $\delta = {}^{4\pi}/_{3}$

Figure 2. Interference patterns obtained by step-by-step phase shift method.

The gamma correction was performed for an each interferogram if it was necessary. To do this the intensity at an each interferogram's point was exponentiated to 1.25, i.e. $I(x,y) = [I(x,y)]^{\gamma}$, where $I(x,y)$ – the intensity of the interferogram point (x,y), $\gamma = 1.25$ – the gamma correction coefficient.

Then intensity vectors are formed at an each image pixel. These intensity vectors contain intensity values at an specific image point at different phase shifts. Let $I_1(x,y)$ - the intensity at the point (x,y) on the first interferogram, $I_2(x,y)$ – the intensity on the second interferogram, $I_3(x,y)$ – the intensity on the third interferogram. Then the intensity vector at image pixel is $\boldsymbol{I}(x,y) = [I_1(x,y) \quad I_2(x,y) \quad I_3(x,y)]$.

Next, the vector orthogonal to the each formed intensity vector is computed. The orthogonal vector can be computed using the matrix equation:

$$\boldsymbol{I}^{\perp} = M \cdot \boldsymbol{I}$$

The transformation matrix must satisfy the following requirements:

$$|M| = 0,$$

$$M \cdot [1 \cdots 1]^T = 0$$

In our case the transformation matrix will be:

$$\mathrm{M} = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

Computed orthogonal vectors will have the same size as the intensity vectors, i.e. they contain three elements. Figure 3 shows the orthogonal vector trajectory in the intensity space. Here coordinates of each point are elements of the corresponding orthogonal vector. A shape and a location of points cloud depends on phase shifts.

Orthogonal vectors properties ensure that all points in the intensity space are located in the same plane. This fact allows to reduce the problem of dimensionality. To do this we rotate the obtained points cloud parallel a coordinate plane, for example XY plane. Then we project the points cloud to the corresponding coordinate plane.

After that we approximate the points cloud using ellipse. Using known analytic geometry formulas we can write the trajectory equation as a second order curve equation [4]:

$$a_{11}x^2 + a_{22}y^2 + 2a_{12}xy + 2a_{13}x + 2a_{23}y + a_{33} = 0$$

To retrieve the coefficients of the second order curve equation the least squares method can be used. However this method can't provide a required measurement accuracy. Therefore to retrieve the ellipse equation coefficients we used the approximation algorithm which is described in [3].

This algorithm is based on a search of solutions, which satisfy the condition $4a_{11} \cdot a_{22} - a_{12} = 1$ marking out an ellipse trajectory in other possible trajectory types which are the conic sections (parabola, hyperbola).

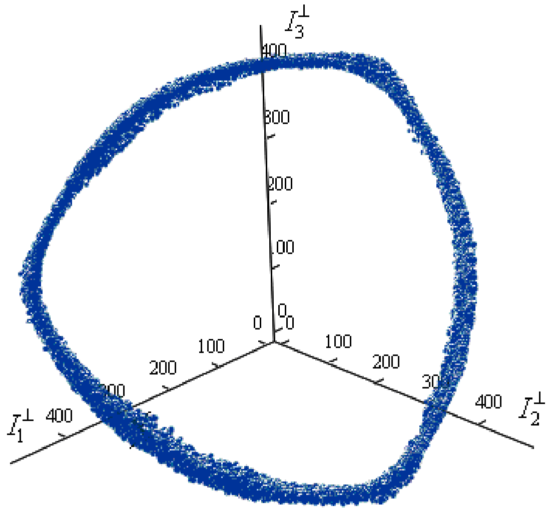


Figure 3. The orthogonal vectors location in the intensity space.

Main steps of this approximation algorithm are following.

1. We form the matrix D, consisting of following elements:

$$D = \begin{bmatrix} x_1^2 & y_1^2 & x_1y_1 & x_1 & y_1 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_i^2 & y_i^2 & x_iy_i & x_i & y_i & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_n^2 & y_n^2 & x_ny_n & x_n & y_n & 1 \end{bmatrix},$$

where x_i and y_i are i -th point coordinates on the plane, $i = 1..n$, n – total number of points.

2. We compute the matrix $S = D^T D$.
3. We solve the generalized eigenvalue problem

$$Sq = \lambda Cq,$$

where a matrix C is

$$C = \begin{bmatrix} 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Here we need to find eigenvalues λ and corresponding them eigenvectors q.

4. We must choose the minimum absolute eigenvalue. An eigenvector corresponding to this eigenvalue will determine ellipse equation coefficients.

Figure 4 shows the points cloud on the coordinate plane and its ellipse approximation.

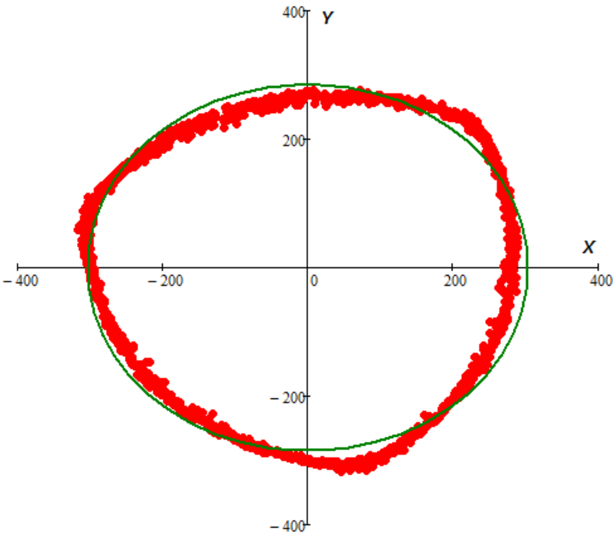


Figure 4. The ellipse approximation of the points cloud.

When the ellipse equation coefficients are found we must move the points cloud center (x_0, y_0) to the coordinate plane origin. For this purpose we calculate the ellipse's center coordinates using the following formulas [4]:

$$x_0 = \frac{\begin{vmatrix} a_{13} & a_{12} \\ a_{23} & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{vmatrix}}, y_0 = \frac{\begin{vmatrix} a_{11} & a_{13} \\ a_{12} & a_{23} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{vmatrix}}$$

Then we subtract these coordinates from the corresponding coordinates of each cloud point. The next step is to rotate the points cloud so that ellipse principal axes coincide with the coordinate axes. The required rotation angle Ω can be calculated using an expression [4]:

$$\Omega = \frac{1}{2} arctg \frac{2a_{12}}{a_{11} - a_{22}}$$

Then all cloud points are corrected to the ellipse. For the each cloud point we get the nearest ellipse point. The nearest ellipse point is an intersection point of the ellipse and the normal from this point to the ellipse.

The final step is to transform the ellipse into the circle (figure 5). After performed transformations the phase φ at each image point can be calculated using following formula:

$$\varphi_i = arctg \frac{y_i}{x_i}, i = 1 \dots N,$$

where x_i and y_i are i -th point coordinates of circle, N is total number of points.

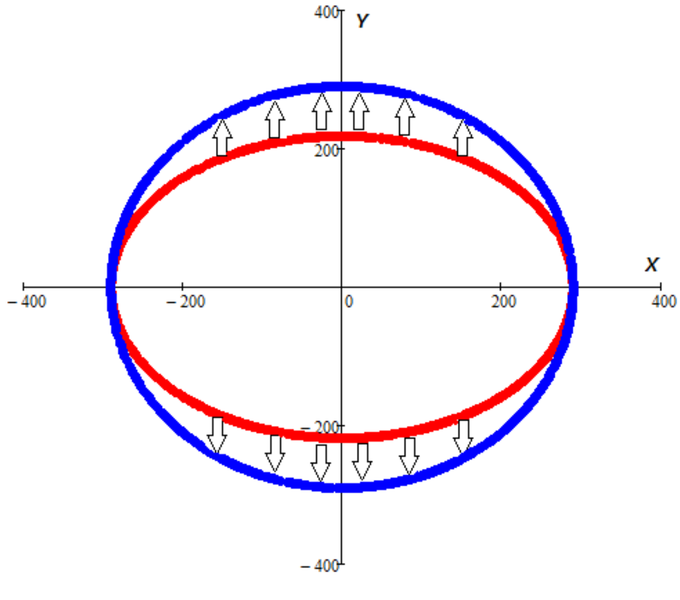


Figure 5. The transformation the ellipse into the circle.

Figure 6 shows the result of phase computing using described above algorithm for interference patterns showed in figure 2. In figure 6 the measured phase is reduced to the range of $0 \dots 255$. Zero phase corresponds to black, 2π phase corresponds to white.

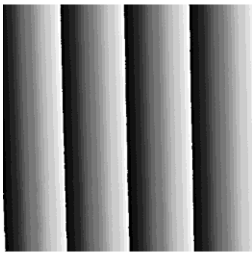


Figure 6. The measured phase field.

Figure 7 shows the graph of measured phase (middle line of image in figure 6) and the graph of phase calculated by decoding formula at known phase shifts. In these graphs the phase is showed in radians. You can see that these graphs are almost identical. Here phase values are wrapped in the range of $0 \ldots 2\pi$. Phase unwrapping procedures are used for further processing. These procedures retrieve the unambiguous phase value out of the wrapped phase.

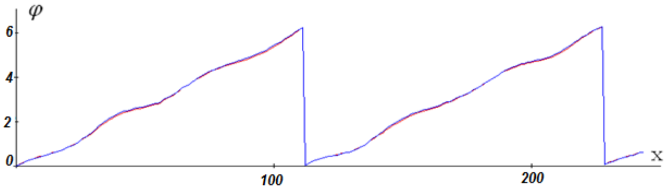


Figure 7. Graphs of measured and calculated phases.

Figure 8 shows the graph of difference between the measured phase and the phase calculated by well-known decoding formulas. The graph shows that the maximum absolute deviation is not more than 0.07 radians.

Thus we propose the new decoding algorithm which does not require a priori shift knowledge in phase shifting interferometry. The accuracy of this algorithm is not worse than accuracy of algorithms using known phase shift values.

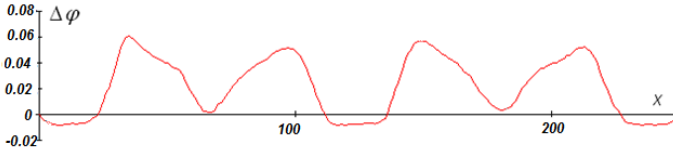


Figure 8. The graph of difference between the measured phase and the calculated phase.

III. Conclusion

This paper describes a new decoding method for interference patterns in which the interference signals trajectory is reduced to the well-known (circular) form and therefore does not require a priori information about the phase shift magnitude. We compared results obtained by the decoding of the interferogram at known phase shifts with the results obtained by the new method. The comparison shows that the proposed interferogram's decoding method is not worse than decoding methods using known phase shift values. Thus the proposed algorithm can be used to construct the interference measurement systems for high accuracy measurements.

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