Step-by-Step method with Phase Shift Is-use Changes in the Intensity of Interfering Beams

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Abstract – We consider the theoretical background to the justification of new approaches to obtaining and analysis, interferograms. The proposed approach is based on a combination of methods for the analysis of interferograms obtained by the intensity change of the interfering beams and the phase shift.

Keywords – Interferogram analysis, phase shift, amplitude.

I. INTRODUCTION

Analysis of interferograms is an important part of digital interference, including holographic measuring systems. The largest application received measurement systems based on step-by-step phase shifts. There are a large number of different algorithms for the analysis of interferograms that use once-private number of phase shifts, e.g., [1]-[5]. These algorithms require the solution of systems of transcendental equations. The authors proposed a generalized method of analysis of interferograms based on phase shifts using an algebraic approach, which allows simply enough to allow formula to an arbitrary number of phase shifts [2]. However, these methods require the constancy of some parameters across the field of the interferogram, which does not allow achieving the potential accuracy of interferometric measurements [6]-[8].

II. STATEMENT OF THE PROBLEM

In general, the equation of the two interfering wave fields has the appearance of

\[ I(x, y) = A_x(x, y)^2 + A_y(x, y)^2 + 2A_x(x, y)A_y(x, y)\cos(\phi(x, y) + \delta(x, y)) \tag{1} \]

where \( A_x(x, y) \) and \( A_y(x, y) \) - the intensity of the interfering object and reference wave fields, and \( \phi(x, y) \) the difference of their phases and \( A_x(x, y), A_y(x, y) \), \( \phi(x, y) = \phi_p(x, y) - \phi_r(x, y), \delta(x, y) \). Insertion phase shift, respectively. When changing the phase shift derived a series of equations. Each of the equations in the General case has four unknown.

To reduce the number of unknowns impose Ogres-limitations on the constancy and certainty of the parameters of equations:

\[ A(x, y) = const, \phi(x, y) = const \] at each point \((x, y)\) the whole series of interferograms, phase shift \( \delta(x, y) \) is considered to be known and constant throughout the field of the interferogram. Given the above, and rewriting a label of variables

\[ A_p(x, y) + A_p(x, y) = A(x, y) \quad \text{and} \quad 2A_p(x, y)\cdot A_p(x, y) = B(x, y) \]

you can get a system of equations where number of equations equals the number of unknowns.

The expression for the i-th equation has the following form

\[ I_i(x, y) = A_i(x, y) + B(x, y)\cos(\phi(x, y) + \delta) \tag{2} \]

In further symbols \((x, y)\) is omitted.

A large number of works in which get the values of the unknown parameters directly from the equations (1,2). This approach significantly reduces the efficiency of the algorithms for the analysis of interferograms [9].

The purpose of this article is to eliminate these constraints by introducing additional equations that can be obtained by changing the intensity of the interfering beams and . Change the intensity of the beam can be, for example, by placing it in neutral density filter. The consistency change of the beam intensity at their minimum cross section considerably simpler technical task than providing a constant phase shift across the field of the interferogram.

The proposed approach will reduce the complexity of the interferometric measurement systems and thus improve the measurement accuracy.

III. THEORY

A. Modification of the basic equations of interferometry

When changing the intensity of the interfering beams \( A_x \) and \( A_y \) for the i-th phase shift equation (1) can be represented as

\[
\begin{align*}
I_i = & \left( A_p \right)^2 + \left( nA \right)^2 + 2A_p\left( sA \cos\theta \right), & i \neq 0 \\
I_i = & \left( A_p \right)^2 + A^2 + 2A_pA \cos\theta, & i = 0
\end{align*}
\tag{3}
\]

where \( r = [k \ p] \) and \( s = [k \ p] \) - the coefficients of changes in the intensity of interfering beams in the case of two phase shifts.

B. The dependence of the mean brightness and amplitude of interfering bands of the amplitude of the beams

In the case of using equations of the form (2) parameters \( A \) and \( B \) depend on the coefficients \( r \) and \( s \) implicitly. Parameter \( B \) depend on \( r \) and \( s \) proportionally, i.e.

\[ B(r, s) = 2\left( rA_p \right)\left( sA_p \right) = rsB \tag{4} \]
where \( B = B(1,1) \).

The dependence of the parameter \( A \) coefficients \( r \) and \( s \) has a more complex (nonlinear) character. Consider the options:

a) changes only the reference beam

\[
A(l,s) = \left( A_p \right)^2 + (sA_d)^2 = \left[ 1 + \frac{B^2}{A^2 + B^2} \right] A, \quad (5a)
\]

where \( A = A(1,1) \).

b) changes only the object beam

\[
A(r,1) = \left( rA_p \right)^2 + (A_d)^2 = \left[ 1 + \frac{A^2}{A^2 + B^2} \right] A. \quad (5b)
\]

c) change both beam

\[
A'(r,s) = \left( rA_p \right)^2 + (sA_d)^2 = \left[ 1 + \frac{A^2}{A^2 + B^2} \right] A. \quad (5c)
\]

C. The Algorithm of calculation of amplitude beams

Further, for ease of explanation, consider a system of equations (1) in vector form [2]:

\[
I = A + (B \otimes C) \cos \phi - (B \otimes S) \sin \phi \quad (6)
\]

where \( \otimes \) - the symbol of Kronecker products of vectors. For example, when the three phase shifts:

\[
\tilde{S} = \begin{bmatrix} \sin \delta_1, & \sin \delta_2, & \sin \delta_3 \end{bmatrix}^T,
\]

\[
\tilde{C} = \begin{bmatrix} \cos \delta_1, & \cos \delta_2, & \cos \delta_3 \end{bmatrix}^T, \quad \text{and} \quad \tilde{I} = [I_0, I_1, I_2]^T.
\]

Then

\[
I \cdot C^T = A \cdot C^T + (B \otimes C)^T \cos \phi - (B \otimes S)^T C^T \sin \phi, \quad (7a)
\]

\[
I \cdot S^T = A \cdot S^T + (B \otimes S)^T \cos \phi - (B \otimes S)^T S^T \sin \phi, \quad (7b)
\]

where \( \tilde{S}^T = \begin{bmatrix} 0 & 1 & 1 \end{bmatrix}, \quad \tilde{C}^T = \begin{bmatrix} 0 & 1 & -1 \end{bmatrix}, \quad \tilde{I} = \begin{bmatrix} 0 & 1 & 1 \end{bmatrix} \).

Note that from the properties of scalar product follows

\[
\tilde{S} \cdot C^T = \tilde{C} \cdot S^T = \tilde{C} \cdot \tilde{S} = 0. \quad (8)
\]

The orthogonal transformation also has the property

\[
\begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} a \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad (9)
\]

where \( a \) - arbitrary constant.

Consider the case when at the same time change the object and reference beam with arbitrary phase values and the phase shift (the index \( j \) is omitted). Let the reference beam is changed in time \( m \) and \( n \), and object \( k \) and \( p \) times, respectively. Then the system of equations of the form (3) can be represented as follows

\[
\begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} A_p^2 + A_d^2 & \cos \theta \cdot \left( kA_d \right)^2 & \cos \theta \cdot \left( mA_d \right)^2 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_p \\ A_d \\ A \end{bmatrix} + \begin{bmatrix} 2 \left( kA_d \right)^2 & 2mA_dA \end{bmatrix} \cos \theta. \quad (10)
\]

Find a vector \( I^\perp \) orthogonal to the vector \( I \)

\[
I_0 = \begin{bmatrix} A_p^2 + A_d^2 \\ (kA_d)^2 + (mA_d)^2 \\ (pA_d)^2 + (nA_d)^2 \end{bmatrix}, \quad I_1 = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} I_0 + \begin{bmatrix} 2 \left( kA_d \right)^2 & 2mA_dA \end{bmatrix} \cos \theta. \quad (11)
\]

Note that taking into account the properties (9) the second term of expression (11) is zero.

Equation (11) given the coefficients of the change of the beam intensity

\[
\begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_p^2 + A_d^2 \\ (kA_d)^2 + (mA_d)^2 \\ (pA_d)^2 + (nA_d)^2 \end{bmatrix}, \quad (12)
\]

Revealing the expression (12) and equating the left and right side we get the system of equations

\[
\begin{bmatrix} m & -n & p \\ k & m & -p \\ n & k & m \end{bmatrix} \begin{bmatrix} A_p^2 + A_d^2 \\ (kA_d)^2 + (mA_d)^2 \\ (pA_d)^2 + (nA_d)^2 \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}. \quad (13)
\]

With known values of \( m, n \) and \( k, p \) by solving the system (13) we can obtain the values \( A_p^2 \) and \( A_d^2 \):

\[
A_p^2 = \frac{kp(mp - kn)I_0 + p(p - n)I_1 + k(k - m)I_2}{(k - m)(n - p)(kp - np)}, \quad (14a)
\]

\[
A_d^2 = \frac{mn(mp - kn)I_0 + n(n - p)I_1 + m(k - m)I_2}{(k - m)(n - p)(np - km)}. \quad (14b)
\]

In case of change of the intensity of only the reference beam assuming \( k = p = 1 \) from (14) we have

\[
A_p^2 = \frac{m - n + (1 - m)I_1 + (1 - m)I_2}{(1 - m)(n - 1)(n - m)}, \quad (15a)
\]

\[
A_d^2 = \frac{mn(m - n)I_0 + n(n - 1)I_1 + m(m - 1)I_2}{(1 - m)(n - 1)(n - m)}. \quad (15b)
\]

Here

\[
A(x, y) = A_p(x, y)^2 + A(x, y)^2
\]

and

\[
B(x, y) = 2(A_p(x, y) \cdot A(x, y)).
\]

After normalization of the equation (2) we obtain an equation
of the form
\[ I(x, y) = \frac{I(x, y) - (A_p(x, y)^2 + A_s(x, y)^2)}{2A_p(x, y)A_s(x, y)} = \cos(\phi(x, y) + \delta). \] 
(15)

Then performed the same calculation with different values of the phase shifts.

The obtained values of \( \tilde{I} \) constitute a system of equations
\[ \tilde{I} = \cos(\phi + \delta), \] 
(16)
from which, you can find and the actual value of insertion phase shifts.

Options with other methods to change the interfering beams is calculated similar to the above case..

D. Calibration coefficients of changes in the intensity of interfering beams

The accuracy of the measurement of the amplitudes of the interfering beams is directly dependent on the true values of the coefficients of transmittance, neutral filters, other devices, which are used to amplitude changes. Therefore, the need for their calibration no doubt. With this goal we consistently place neutral density filters in General, the branch beams to the beam splitter. Consider the process of calibration for a couple and filters with transmission coefficients of \( m \) and \( n \). In this case the system of equations (13) takes the form
\[ \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} I_0 \\ m \hat{I} \\ n \hat{I} \end{bmatrix} = \begin{bmatrix} \frac{m^2}{I_0} & \frac{1}{m} \\ \frac{1}{m} & \frac{m}{I_0} \\ \frac{1}{n} & \frac{1}{n} \end{bmatrix} \begin{bmatrix} \Delta A_p^2 + \Delta A_s^2 \\ (m\Delta A_p)^2 + (m\Delta A_s)^2 \\ (n\Delta A_p)^2 + (n\Delta A_s)^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}. \] 
(17)

Revealing the system (18) will receive
\[ \begin{bmatrix} I_z - \frac{I_0}{m^2} & I_0 - \frac{I_z}{m^2} & \frac{I_z}{n^2} - I_0 \end{bmatrix} \begin{bmatrix} r \\ m \\ n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}. \] 
(18)

Here \( m = \sqrt{\frac{I_z}{I_0}} \) and \( n = \sqrt{\frac{I_z}{I_0}} \).

VI. CONCLUSION

Presents a new method for the determination of the amplitudes of the interfering beams for two-beam interferometry. Used step-by-step method phase shift, where a series of interferograms by introducing a phase shift in the reference arm of the interferometer.

A system of equations, which is the phase difference of the interfering wavefronts, carrying information about the parameters of the measured object. This approach requires persistence parameters of the system of equations, and, consequently, the measuring systems that are difficult to secure. This leads to a reduction of measurement quality.

To eliminate this drawback it is necessary to increase the number of unknowns in the system equations. With the goal of more mutually-independent equations is proposed to modify the amplitude of the reference and (or) object beams. A method of calibration of the coefficients of amplitude changes. Note that in contrast to the classical methods of analysis of interferograms [10, 11] in this case reduces the requirements for the constancy of the mean brightness and amplitude of the interference fringes in a series of interferograms, which leads to an increase of measurement quality.

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REFERENCES


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