The Recovery of the Phase Information from the Digital Holograms with Small Angles of Interference

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***Abstract* – this paper proposes a method of reconstruction of digital holograms with small angles between the interfering wave fields. The proposed approach is based on a combination of methods of digital holography and step-by-step method of phase shifts and allows to obtain not only the amplitude but also the phase information of the measured object.**

***Keywords* – Digital hologram, phase shift, amplitude, phase.**

I. ВВЕДЕНИЕ

Holograms formed by the interference of the object beam reflected from the studied object and the reference wave. From the classical interference patterns, they differ in that the object beam is reflected from objects with a diffuse surface. In this case, the distance from the object to the detection plane with the object wave takes place a transformation, which under certain approximations can be described by the Fresnel transform or the Fourier.

In the original optical scheme of Gabor [1], which simulated the results of the interference of electron beams, reference and object wave are located along the axis normal to the photographic plate. When you restore this leads to a superposition of 0 and ±1 diffraction orders. To allocate from the holograms of the source distribution of the object wave was not possible. Zero bundle is an interruption in the restored signal, the ±1 diffraction orders of the imaginary and will form a real image of the object.

The wide spread of holography was obtained when using off-axis recording scheme, proposed by E. Leith and J. Upatnieks [2]. In off-axis scheme reference and object beams interfere under a certain angle . The greater the angle, the greater the separation of the 0 and ±1 diffraction orders, and the higher quality of the image of the object can be obtained. In holography commonly used angles of about 30 degrees. The use of large angles between the interfering beams is a major shortcoming, which prevented the wide introduction of holographic systems. This is due to high resolution requirements of materials for recording holograms.

Consider the interference of plane waves. The distance between the peaks of the interference fringe can be found from the expression

$$\Delta x = \frac{\lambda}{2\sin(\theta/2)} . \qquad (1)$$

In this case, when $\theta = 30^o$ and $\lambda = 0{,}6\ мкм$ - $\Delta x \approx 1\ мкм$.

By theorem Wittecarra-Kotelnikov-Shannon (WKS) requires at least two points on the period of the strip, i.e. the resolution of the material must provide not less than 0.5 μm, or 2000 lines per mm for the registrations of the carrier frequency.

Methods of digital holography based on the register-tion of holograms on a matrix of photodetectors, and further interpretation by computer systems. R. Laurence and D. Goodman first proposed the digital method of restoration in 1967 [3]. A great contribution to the area of application of digital holography introduced by L. Yaroslavsky and N. Merzlyakov. In his book [4] they give a detailed theoretically, the determination-ical basis conversion, representation of optical wave fields, and describe algorithms based on the discrete Fourier transform. Their ideas were developed by L. Oorala and P. Scott, and moved to the measurement area of the particle in 1987 [5].

Digital image sensors used in digital holo-graphy, are unable at the moment to provide such a high spatial resolution. Therefore, when the digital holographic reconstruction is necessary to reduce the angle between the interfering wave fields.

For small angles, expression (1) takes the form $\Delta x \approx \frac{\lambda}{\theta}$. From this expression we can determine the maximum angle between the interfering waves, which is not-mere frequency may be allowed (taking into account the theorem of the WKS). This is possible at an angle equal to $\theta_{\max} = \frac{\lambda}{2\Delta x}$.

For example, when the pixel size Δx is 5 microns, the maximum angle of the interference can be $\theta_{\max} \approx 3.4^0$ at a length of λ is equal to 0.6 μm. The use of small corners inevitably leads to overlap of the spectra in different diffraction orders, which leads to a distortion of the phase information needed to create a high-precision measuring systems.

The aim of this work is to investigate methods of improving the accuracy of reconstructing the phase profile of the wave front based on the methods step-by-step phase shift at small angles between the interfering beams.

II. THE ANALYSIS OF DIGITAL HOLOGRAMS STEP-BY-STEP METHOD PHASE SHIFT

Methods step-by-step phase shift (phase shifting interferometry - PSI) was widely used for receiving and decoding the classical interference patterns. This is due to the simplicity of the job the individual values of phase shift, simple enough algorithms and the high precision decoding. Step-by-step method phase shift based on the detection of multiple interference patterns while changing the phase of the reference wave at the known value. The phase shift between the interfering beams can be realized in different ways. The most simple phase shift to define the movement of the mirror mounted on the piezoelectric ceramics.

Depending on the number of phase shifts there are different decryption algorithms. In [7-9] we obtained a generalized scheme of the algorithm for various numbers of shifts. If we can register m of the interference patterns with phase shifts δ_i (*i* = 0,1,2, ... , *m-1*), where m – the number of phase shifts, $\delta_0 = 0$:

$$I_i(x,y) = A_p^2(x,y) + A_r^2(x,y) + \\ +2A_p(x,y)A_r(x,y)\cos\left(\varphi_p(x,y) - \varphi_r(x,y) + \delta_i\right). \quad (2)$$

This expression can be represented in the form

$$I_i(x,y) = A(x,y) + B(x,y)\cos(\phi(x,y) + \delta_i), \quad (3)$$

The phase difference $\phi(x,y) = \varphi_p(x,y) - \varphi_r(x,y)$ between the reference and object beams may be determined as

$$\phi = \arctan\left(\frac{\vec{I}^{\perp}\cdot\vec{C}}{\vec{I}^{\perp}\cdot\vec{S}}\right), \quad (4)$$

where $\vec{C} = (\cos\delta_0, \ldots, \cos\delta_{m-1})^T$, $\vec{S} = (\sin\delta_0, \ldots, \sin\delta_{m-1})^T$, and $\vec{I}^{\perp}$ - orthogonal vector, such that $\left(\vec{I}\cdot\vec{I}^{\perp}\right) = 0$.

The orthogonal vector can be found using the matrix equation

$$\vec{I}^{\perp} = M\cdot\vec{I}. \quad (5)$$

When multiple dimensions (*m*) matrix M will have a look

$$M = \begin{bmatrix} 0 & 1 & 0 & \ldots & 0 & -1 \\ -1 & 0 & 1 & \ldots & 0 & 0 \\ 0 & -1 & 0 & \ldots & 0 & 0 \\ \vdots & \vdots & \vdots & O & \vdots & \vdots \\ 0 & 0 & 0 & \ldots & 0 & 1 \\ 1 & 0 & 0 & \ldots & -1 & 0 \end{bmatrix}_{m\times m}. \quad (6)$$

Knowing the phase difference and the phase of the reference wave $\varphi_r(x,y)$ to determine the initial phase distribution $\varphi_p(x,y)$.

Unlike classic interferometry, where the main objective of decoding is to obtain the phase difference between the interfering beams, we also need to determine the phase and amplitude of the original wave. Most simply it can be done, if the reference beam to use a flat wave with constant amplitude. And value *B* can be defined as

$$B(x,y) = 2A_p(x,y)A_r(x,y)$$

$$B = \frac{1}{\left|\vec{S}\cdot\vec{C}^{\perp}\right|}\sqrt{\left(\vec{I}\cdot\vec{S}^{\perp}\right)^2 + \left(\vec{I}\cdot\vec{C}^{\perp}\right)^2}, \quad (7)$$

If you know the amplitude of the reference field to $A_r(x,y)$, knowing $B(x,y)$ can be identified and $A_p(x,y)$. Thus it is possible to determine the amplitude $A_p(x,y)$ and phase $\varphi_p(x,y)$ of the initial wave front in the plane of the hologram.

To restore the original wave front it is possible to use a Fresnel transform or a Fourier transform, with which you can restore an exact copy of the original wave reflected from the object.

III. THE PROPOSED METHOD

Most of the information about the object enclosed in phase. Consider how the reconstruction of phase objects with small angles of shear.

Плоскость (η,ξ) - the plane of formation of the hologram. For conversion into the plane of the hologram depending on the distance *d* you need to make a Fresnel transform or Fourier. For the formation of a hologram, you need to add the reference wave front.

$$U_r(i,j) = a_r\exp\left(i\frac{2\pi}{\lambda}(\sin\theta_x\cdot\frac{X_{\max}}{N_x}i + \sin\theta_y\cdot\frac{Y_{\max}}{N_y}j)\right), \quad (8)$$

where $X_{\max}\times Y_{\max} = 10мм\times 10мм$, the angles of inclination of the normal to the recording plane: $\theta_x = 0.9^{\circ}$, $\theta_y = 0.9^{\circ}$. The amplitude of the sum of the two wave fronts is proportional to the intensity and can be considered as a hologram.

The plane (x', y') is the observation plane. To obtain an image of the object produced by a Fresnel transform or a Fourier transform on the hologram depending on the distance . d' In the image plane, we can determine both the amplitude and phase of the original object. Unfortunately, the phase information at the small angles of the reference beam is strongly distorted. Methods based on filtering [6], give good results in the recovery of the amplitude, but the phase information does not improve.

We propose a new method based on step-by-step phase shift.

Take also the distribution of complex amplitudes. In the plane of formation of the hologram we will add support up front with known phase shift.

$$U_i(i,j) = a_r\exp\left(i\left[\frac{2\pi}{\lambda}(\sin\theta_x\cdot\frac{X_{\max}}{N_x}i + \sin\theta_y\cdot\frac{Y_{\max}}{N_y}j) + \delta_i\right]\right), \quad (9)$$

where δ_i (*i* = 0,1,2, ... , *m-1*) – known phase shifts, *m* – the number of phase shifts. Let $m = 4$.

Then we can form the hologram 4. According to the algorithm described in the previous paragraph, we shape the phase and amplitude of a complex wave field of the object. In the image plane will receive a distribution of the recovered phases. The phase distribution has a slope caused by tilting of the beam during formation of holograms, since the phase difference is equal to $\phi(x,y)=\varphi_p(x,y)-\varphi_r(x,y)$. The slope will be absent at the zero angle. Since $\varphi_r(x,y)=0$, the phase difference is equal to $\phi(x,y)=\varphi_p(x,y)$.

To remove the influence of slope in two ways. Subtract the known values for the phase distribution of the reference field in the image plane of the hologram or multiply artificially generated wave front in the plane of the hologram to the appropriate reference wave. In this case, distortion of the wave front are eliminated.

IV. CONCLUSION

We propose a new method of decoding holograms of phase objects through a step-wise phase shift. Unlike existing methods described in the literature [10], is used to decode not only restored by the method of PSI phase but also the amplitude. The algorithm of determination of the amplitude distribution of the reference wave.

This method works well with a small angle between the interfering waves and the Central reference beam (Gabor hologram) that allows the use of digital image sensors with small spatial resolution at registration of holograms.

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REFERENCES

[1] De Groot P. ,“101-frame algorithm for phase shifting interferometry,” // Proc. SPIE. 1997. 3098. P. 283–292.

[2] V.I. Guzhov, S.P. Ilinykh, R.A. Kuznetsov, D.S. Haydukov, “Generic algorithm of phase reconstruction in phase-shifting interferometry,” Opt. Eng. 2013, 52(3):030501.

[3] S. Suriyasirikun, S.T. Khlayboonme, W. Thowladda, "Phase-Shifting Interferometry for Surface Roughness Measurement on Glass Substrates", Advanced Materials Research, Vol. 979, pp. 463-466, 2014.

[4] V.I. Guzhov, S.P. Il’yinykh, I.A. Sazhin, E.N. Denezhkin, E.S. Kabak, and D. S. Khaidukov, “Quasiheterodyne method of interference measurements,” Optoelectronics, Instrumentation and Data Processing. - 2015. - Vol. 51, iss. 3. - P. 280-286.

[5] Wengierow, M., Salbut, L., Ramotowski, Z., Szumski, R., Szykiedans, K. Measurement System Based on MultiWavelength Interferometry for Long Gauge Block Calibration Metrology and Measurement Systems. - 2013. - Volume XX, Issue 3, p. 479–49.

[6] V.I. Gushov, S.P. Ilinykh, D.S. Haidukov, R.A. Kuznetsov, “Estimation of validity of optical measurements,” Proceedings of APEIE 2012. 11-th International Conference on actual problem electronics instrument engineering. vol. I , 2012, pp. 146–149.

[7] V.I. Guzhov, S.P. Il’yinykh, D.S. Khaidukov and A.R. Vagizov, “Eliminating phase-shift errors in interferometry, ”Optoelectronics, Instrumentation and Data Processing.-2011., vol. 47, Nu.1, pp. 76-80.

[8] V.I. Gushov, S.P. Ilinykh, D.S. Haidukov, R.A. Kuznetsov , “Method of an Assessment of Reliability of High-Precision Measurements,” Proceedings of APEIE 2012. 11-th International Conference on actual problem electronics instrument engineering. vol. I, 2012, pp. 105–106.

[9] Quiroga JA, Gómez-Pedrero JA, García-Botella A "Algorithm for fringe pattern normalization," Opt Communications, 2001, 197:43.

[10] J. C. Wyant, “Interferometric optical metrology: basic system and principles,” Laser Focus 18(5), 65–67 (1982).

[11] Servín, Manuel,, J. Antonio Quiroga, and J. Moises Padilla. Fringe Pattern Analysis for Optical Metrology: Theory, Algorithms, and Applications. Weinheim, Germany: Wiley-VCH Verlag GmbH & Co. KGaA, 2014.

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