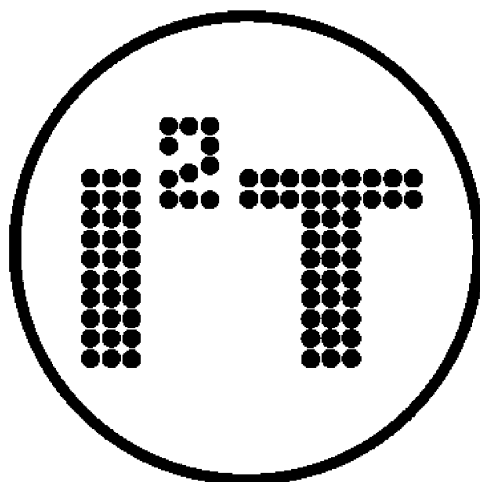


**International Scientific – Practical Conference  
«INNOVATIVE INFORMATION  
TECHNOLOGIES»**



**PART 3  
INNOVATIVE INFORMATION TECHNOLOGIES  
IN INDUSTRY AND SOCIAL-ECONOMIC SPHERE**

**Prague – 2014  
April 21-25**

observed value is unknown, as well as the ease of calculations performed on a personal computer. Bootstrap approach is quite convenient to perform calculations based on resampling and statistical estimation. It is particularly effective when there is a limited number of data to make forecasts about the processes or systems behavior.

The matters of the bootstrap application for the reliability analysis, electronics products particularly, are still relevant, as evidenced by the continued emergence of new papers on the subject.

### References

- 1 Efron B. Bootstrap Methods: Another Look at the Jackknife. // *Ann. Statist.* – 1979. – Vol. 7, N 1. – p. 1-26.
- 2 Shitikov, V.K. and Rosenberg, G.S. Randomization and bootstrap: statistical analysis at biology and ecology using R. - Togliatti: Cassandra, 2013. – 314 p.
- 3 Mosteller, F., Tukey J. Data analysis and regression. M: Finance and statistics, 1982. MY. 1. – 320 p.
- 4 Adler, Y.P. et al. Bootstrap simulation in the construction of confidence intervals for censored samples. // *Factory laboratory.* - 1987. - № 10. - p. 90-94.
- 5 Adler, Yu.P. et al. The use of the bootstrap method for determining lower confidence estimating durability. // *Reliability and quality control.* - 1987. - № 9. - P.50-54.
- 6 Adler, Yu.P. et al. Application of the bootstrap method for complex product resource forecasting with regard to expert estimates. - *Reliability and quality control.* - 1988. - № 8. - P.29-33.
- 7 Adler, Yu.P. et al. Forecasting the expert assessments of technical condition using the bootstrap. - *Reliability and quality control.* - 1989. - № 12. - P. 13-21.
- 8 Seppala, T., Moskowitz, H., Plante, R., and Tang J. Statistical Process Control via the Subgroup Bootstrap. // *Journal of Quality Technology.* – 1995. - V. 27, P. 139-153.
- 9 Liu, R.Y. and Tang, J. Control Charts for Dependent and Independent Measurements Based on the Bootstrap. // *Journal of the American Statistical Association.* – 1996. - V. 91. – P. 1694-1700.
- 10 Jones, I.A. and Woodall, W.H. The Performance of Bootstrap Control Charts. // *Journal of Quality Technology.* – 1998. – V. 30. – No. 4. – P. 362 – 375.
- 11 Lio, Y. L. and Park, C. A Bootstrap Control Chart for Birnbaum-Saunders Percentiles. *Quality and Reliability Engineering International.* – 2008. – V. 24. – P. 585 - 600.
- 12 Park, H.I. . Median Control Charts Based on Bootstrap Method. *Communications in Statistics – Simulation and Computation.* – 2009. – V. 38. – P. 558-570.
- 13 Kuznetsov L.A., Zhuravlev M.G. Mapping quality control using non-parametric Wilcoxon test - Mann - Whitney. // *Factory laboratory.* - 2009. - № 1. - p. 70-74.

### QUASI-HETERODYNE METHOD OF THE PHASE MEASURING IN A SERIES OF INTERFEROGRAMS

Guzhov, V., Ilinykh, S., Sazhin, I., Kabak, E.  
*Novosibirsk, Novosibirsk State Technical University*

This paper proposes a method for measuring the phase difference based on the analysis of Lissajous figures formed intensities of pairs of points in a series of interferograms with different phase shifts. This method does not require the definition of the actual values of the phase shifts.

Keywords: interferogram, phase shift, Lissajous figures, interferometry

### 1. THE PROBLEM

The aim of this work is an explicit determination of the phase difference on the intensity values of two arbitrary points in the interferogram using the method of step phase shift [1-5]. Moreover, in the present time a lot of attention paid to the analysis of the measurement errors caused by inaccuracy of the phase shifts [6-11]. Equation intensities at each point of the interferogram at different values of insertion phase shift can be written as [3]:

$$I_i(x, y) = A(x, y) + B(x, y) \cos[\phi(x, y) + \delta_i], \quad (1)$$

where  $A(x, y)$  - average brightness,  $B(x, y)$  - the amplitude of the interference fringes.

May be assumed that all points  $(x, y)$  phase shifts are the same. This assumption is satisfied in most cases on the basis of physical conditions of the experiment. Then we can get the additional equation, considering the decisions are not in one but in several spatial points interferogram.

$$I_i(x_k, y_k) = A(x_k, y_k) + B(x_k, y_k) \cos[\phi(x_k, y_k) + \delta_i], \quad (2)$$

### 2. DESCRIPTION OF THE METHOD

If you take two arbitrary points on the interferogram with coordinates  $A(x_A, y_A)$  and  $B(x_B, y_B)$ , then the five phase shifts we obtain a system of ten equations of the form (2) with ten unknowns.

To simplify notation the intensity change in the system of equations (2)  $I_{1A} \dots I_{5A}$  as  $x_1 \dots x_5$ , intensity and  $I_{1B} \dots I_{5B}$  as  $y_1 \dots y_5$  and the average brightness levels at the points and  $x_0$  and  $y_0$ , respectively. Taking into account the adopted notation system of equations (2) takes the form

$$x_i = x_0 + B_1 \cos(\phi_1 + \delta_i), \quad y_i = y_0 + B_2 \cos(\phi_2 + \delta_i), \quad (3)$$

here  $i=0, 1 \dots 4$ ,  $\delta_0 = 0$ .

The system of equations (3)  $(x, y)$  can be regarded as the coordinates of points on a plane (fig. 1). Point 1 corresponds to a phase shift  $\delta_0 = 0$ . If the condition  $\delta_0 < \delta_1 < \delta_2 \dots < \delta_4$  is at this point 1 to point 2 switches, etc. This condition is required for the formation of a consistent trajectory of the point 1. For arbitrary phase shifts the trajectory of the point 1 would be move a chaotically. In this case, to ensure a consistent trajectory points can be sorted using the conditional vector formed by the difference between the coordinates of points and the coordinates of the center of the conditioned which is found by averaging the coordinates of all the points. Conditional vectors are compared with each other values of the angles which they form between themselves.

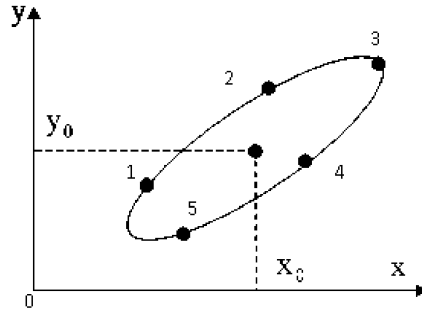


Figure 1. Coordinates of points on a plane

Any point corresponding to the system of equations (3) belongs to a well-known ellipse as Lissajous figure (addition of two sinusoidal waves of the same frequency). Then the system of equations (3) can be rewritten as follows:

$$x_i = x_0 + B_1 \cos(\delta_i), \quad y_i = y_0 + B_2 \cos(\delta_i + \phi_2 - \phi_1), \quad (4)$$

here  $\phi_2 - \phi_1$  - the phase difference between two different points in the interferogram

Lissajous ellipse equation corresponding to the system of equations (4) has the form

$$\frac{(x_i - x_0)^2}{B_1^2} + \frac{(y_i - y_0)^2}{B_2^2} - 2 \frac{(x_i - x_0)(y_i - y_0)}{B_1 B_2} \cos(\phi_2 - \phi_1) = \sin^2(\phi_2 - \phi_1) \quad (5)$$

However, the direct determination of the coefficients of equation (5) encounters great difficulties because, as a rule, it is necessary to find the unknown parameters of average brightness and amplitudes or their evaluation [12]. On the other side of the general equation of an ellipse can be represented as follows [13]:

$$a_{11}x^2 + 2a_{12}xy + a_{22}y^2 + 2a_{13}x + 2a_{23}y + a_{33} = 0. \quad (6)$$

Given the invariance of the leading coefficients of the equation (6) to the plane of the ellipse parallel transport module can be the cosine of the phase difference  $\phi_2 - \phi_1$  of the coefficients of the following equation (6)  $a_{11} = \frac{1}{B_1^2}$ ,  $|a_{12}| = \left| \frac{1}{B_1 B_2} \cos(\phi_2 - \phi_1) \right|$  and  $a_{22} = \frac{1}{B_2^2}$ .

We compute the coefficients listed equation (6) using the equation of the pencil of curves of the second order [14]:

$$f_1(x, y) \cdot f_2(x, y) = \alpha f_3(x, y) \cdot f_4(x, y), \quad (7)$$

here  $f_i(x, y) = A_i x + B_i y + C_i$  - is the equation of a straight line, and  $\alpha$  is selected so that the curve is selectable from a beam passing through a free point 3 (see Figure 2).

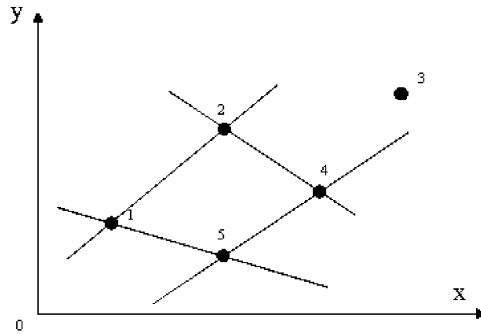


Figure 2. The procedure for connecting the points forming the beam curves

Then

$$A_1 = y_2 - y_1, \quad B_1 = x_1 - x_2, \quad C_1 = x_2 y_1 - x_1 y_2,$$

$$A_2 = y_5 - y_4, \quad B_2 = x_4 - x_5, \quad C_2 = x_5 y_4 - x_4 y_5,$$

$$A_3 = y_5 - y_1, \quad B_3 = x_1 - x_5, \quad C_3 = x_5 y_1 - x_1 y_5,$$

$$A_4 = y_4 - y_2, \quad B_4 = x_2 - x_4, \quad C_4 = x_4 y_2 - x_2 y_4,$$

$$\alpha = -(A_1 x_3 + B_1 y_3 + C_1)(A_2 x_3 + B_2 y_3 + C_2) / [(A_3 x_3 + B_3 y_3 + C_3)(A_4 x_3 + B_4 y_3 + C_4)]$$

and we obtain

$$a_{11} = A_1 A_2 + \alpha A_3 A_4, \quad a_{12} = \frac{1}{2} [A_1 B_2 + A_2 B_1 + \alpha (A_3 B_4 + A_4 B_3)], \quad a_{22} = B_1 B_2 + \alpha B_3 B_4, \quad (8)$$

In our case the module is equal to the phase difference  $|\phi_2 - \phi_1| = \left| \arccos \left( \frac{|a_{12}|}{\sqrt{|a_{11}|} \sqrt{|a_{22}|}} \right) \right|$ , is

defined in the range from 0 to  $\frac{\pi}{2}$ , for determining the true value of the phase difference in the

range from 0 to  $2\pi$  is also necessary to determine the quadrant in which the angle  $\phi_2 - \phi_1$ . Practically, this can be done by tracking the orientation of the main axis of the ellipse and the direction of change in the trajectory of point 1 at different phase shifts (Fig. 1).

### 3. CONCLUSION

This paper proposes a new method for determining the phase difference of two arbitrary points of the interferogram. Fixing one of the points can determine the distribution of the phase-field over the field. Using this method allows to reduce the requirements for setting accuracy insertion phase shifts can be used as measuring systems with a priori unknown insertion phase shifts.

### References

1. Hariharan P., Oreb B.F., Brown N. Digital phase-measurement system for real-time holographic interferometry // Optics Communication.- Vol.41.- №6.-1982.- pp.393-398
2. Wyant J.C., Creath K. Recent advances in interferometric optical testing // Laser Focus.- 1985.- pp.118-132.
3. Wyant J.C. Interferometric optical metrology: basic system and principles // Laser Focus.- 1982.- pp.65-67.
4. Creath K. Phase-shifting speckle interferometry // Applied Optics. 1985. V.24. P.3053–3058.
5. J.E.Greivenkamp and J.H.Bruning, “Phase shifting interferometry,” in Optical Shop Testing, Ed. by D.Malacara (Wiley, New York, 1992), Chapter 14, pp. 501–598.
6. P. de Groot. Phase-shift calibration errors in interferometers with spherical Fizeau cavities // Applied Optics.-1994.-V.34.-No.16.-pp.2856-2863.
7. P. de Groot. 101-frame algorithm for phase shifting interferometry. EUROPTO, 1997, Preprint 3098-33.
8. J. Millerd, N. Brock, J. Hayes, et al., “Modern Approaches in Phase Measuring Metrology,” Proc. SPIE. 5856, 14–22 (2004).
9. P. Gao, B. Yao, N. Lindlein, et al., “Phase-Shift Extraction for Generalized Phase-Shifting Interferometry,” Opt.Lett., 2009, 34 (22), 3553–3555.
10. V.I. Guzhov, S.P. Il'yinykh, D.S. Khaidukov and A.R. Vagizov Eliminating phase-shift errors in interferometry // Optoelectronics, Instrumentation and Data Processing.- 2011., Vol.47, Nu.1.- pp. 76-80
11. Schmit J., Creath K. Extended averaging technique for derivation of error-compensating algorithms in phase-shifting interferometry. // Applied Optics.-1995.-V.34.-No.19.-pp.3610-3619.
12. Brad Kimbrough, Neal Brock, James Millerd «Dynamic surface roughness profiler» SPIE Vol. 8126
13. Korn, G. A. and Korn, T. M., Mathematical Handbook for Scientists and Engineers, Second Edition, Dover, New York, 2000.
14. Alexandrov P.S. Lekcii po analiticheskoi geometrii. Moskow: Nauka., 1968. - 912 c.

The financial support provided by the Russian Foundation for Basic Research (RFBR) under grant 14-08-01100 is gratefully acknowledged.