ISSN 8756-6990, Optoelectronics, Instrumentation and Data Processing, 2011, Vol. 47, No. 1, pp. 76–80. © Allerton Press, Inc., 2011.
Original Russian Text © V.I. Guzhov, S.P. Il'yinykh, D.S. Khaidukov, A.R. Vagizov, 2011, published in Avtometriya, 2011, Vol. 47, No. 1, pp. 96–101.

OPTICAL INFORMATION TECHNOLOGIES

Eliminating Phase-Shift Errors in Interferometry

V. I. Guzhov, S. P. Il'yinykh, D. S. Khaidukov, and A. R. Vagizov

Novosibirsk State Technical University,
pr. Karla Marksa 20, Novosibirsk, 630092 Russia
E-mail: vig@nstu.ru

Received November 17, 2010

Abstract—A noiseless algorithm for analyzing phase-shifting interferograms is proposed.

DOI: 10.3103/S8756699011010110

Keywords: optics, interferometry, incremental phase shifting.

INTRODUCTION

Recently, algorithms for recording and decoding phase-shifting interferograms (phase-shifting interferometry) have found increasing use in designing interference systems [1–4]. This is due to the ease of setting the phase shift, simple algorithms, and high-precision decoding, with existing interferometer designs being easily adaptable.

The incremental phase shifting method is based on recording several interferograms with a change in the phase of the reference wave by known values. The accuracy of phase measurement depends on the setting of the value of the applied phase shifts, but in practice, its exact value is difficult to determine due to errors of phase-shifting devices [5]. Therefore, calibration operations are required before each series of measurements.

The objective of this work is to improve the accuracy of interpretation of interferograms.

METHOD AND ITS PRACTICAL IMPLEMENTATION

The essence of the proposed method is to determine the real value of the applied phase shifts by analyzing the trajectory of the hodograph of interference signals (hereinafter, the trajectory of interference signals) at two arbitrary points (A and B) in the interferograms.

The intensity of interferograms s with a phase shift δ_i an be represented as

$$I_i(x,y) = I_0(x,y)[1 + V(x,y)\cos(\phi(x,y) + \delta_i)], \tag{1}$$

where $i = 0, 2, \ldots, m-1$ (m is the number of phase shifts) and $\delta_0 = 0$.

The main purpose of decoding is to determine the phase difference between the interfering wave fronts $\phi(x,y)$ using the recorded intensities $I_i(x,y)$.

It can be assumed that the phase shifts at neighboring points are identical. From the physical conditions of experiments, this assumption is valid in most cases. Then, considering solutions at several, rather than one, points (x_k, y_k), we obtain the additional equation

$$I_{i,\,k}(x_k, y_k) = I_{0,\,k}(x_k, y_k)[1 + V_k(x_k, y_k)\cos(\phi(x_k, y_k) + \delta_i)],$$

or

$$I_{i,\,k} = I_{0,\,k}[1 + V_k\cos(\phi_k + \delta_i)]. \tag{2}$$

In general, the number of points is $k = 1, \ldots, n$, the number of unknowns is $n3 + m - 1$, and the number of equations is $n(m-1)$. The problem is solved if the number of equations is greater than, or equal to, the number of unknowns, i.e.,

$$nm \geq 3n + (m-1). \tag{3}$$

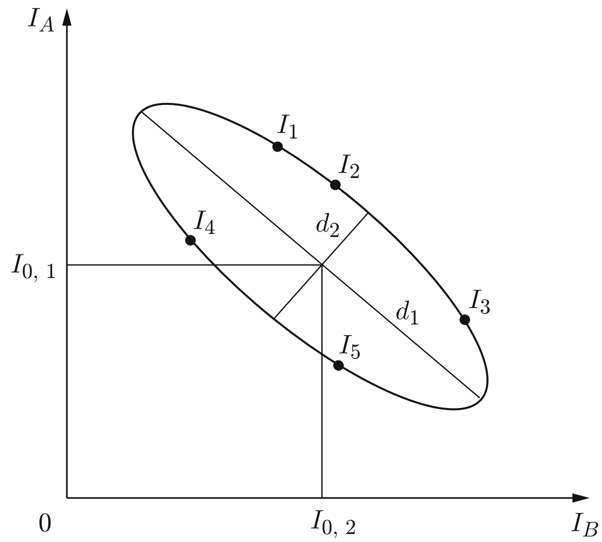

Fig. 1.

An analytical solution is found in recording five interferograms with phase shifts $\delta_0 = 0, \delta_1, \delta_2, \delta_3, \delta_4$. As a result, we have 10 transcendental equations with 10 unknowns $(I_{0,\,1}, I_{0,\,2}, V_1, V_2, \phi_1, \phi_2, \delta_1, \delta_2, \delta_3, \delta_4)$.

In this paper, we consider a numerical method for determining phase shifts from the values of interference patterns at two points:

$$I_{i,\,1} = I_{0,\,1}[1 + V_1 \cos(\phi_k + \delta_i)]; \quad I_{i,\,2} = I_{0,\,2}[1 + V_2 \cos(\phi_k + \delta_i)], \tag{4}$$

where $i = 0, 2, \ldots, m-1$ $(m \geq 5)$.

We will seek a solution in the complex plane with the axes I_1 and I_2 of the intensities of the first and second points corresponding to different phase shifts. As the shift angles are varied from 0 to 2π, the point on the complex plane describes a certain trajectory (Fig. 1). The trajectory of interference signals in the space of intensities is a central curve of the second order — an ellipse.

To determine the characteristics of the trajectory, it is necessary to find the coefficients of the equation of the approximating curve [6]

$$a_{11}x^2 + a_{22}y^2 + 2a_{12}xy + 2a_{13}x + 2a_{23}y + a_{33} = 0, \tag{5}$$

where $x = d_1 \cos(\phi_1)$; $y = d_2 \cos(\phi_2)$ ($d_1 = I_{0,1}V_1$ and $d_2 = I_{0,1}V_2$ are the main axes of the ellipse).

In view of the redundancy of the initial data, it is reasonable to calculate the coefficients of expression (5) using the least squares method:

$$\begin{bmatrix} a_{11} \\ a_{12} \\ a_{22} \\ a_{13} \\ a_{14} \\ a_{33} \end{bmatrix} = \begin{bmatrix} N & \sum_i x_i & \sum_i y_i & \sum_i x_i y_i & \sum_i x_i^2 & \sum_i y_i^2 \\ \sum_i x_i & \sum_i x_i^2 & \sum_i x_i y_i & \sum_i x_i^2 y_i & \sum_i x_i^3 & \sum_i x_i y_i^2 \\ \sum_i y_i & \sum_i x_i y_i & \sum_i y_i^2 & \sum_i x_i y_i^2 & \sum_i x_i^2 y_i & \sum_i y_i^3 \\ \sum_i x_i y_i & \sum_i x_i^2 y_i & \sum_i x_i y_i^2 & \sum_i x_i^2 y_i^2 & \sum_i x_i^3 y_i & \sum_i x_i y_i^3 \\ \sum_i x_i^2 & \sum_i x_i^3 & \sum_i x_i^2 y_i & \sum_i x_i^3 y_i & \sum_i x_i^4 & \sum_i x_i^2 y_i^2 \\ \sum_i y_i^2 & \sum_i x_i y_i^2 & \sum_i y_i^3 & \sum_i x_i y_i^3 & \sum_i x_i^2 y_i^2 & \sum_i y_i^4 \end{bmatrix}^{-1} e, \tag{6}$$

where e is a unit vector with dimension equal to the number of coefficients of the equation.

The trajectories of the interference signals in the space of intensities are presented in Fig. 2; the points show the trajectory obtained by applying phase shifts, and the solid curve is the result of its approximation by a second-order polynomial (5).

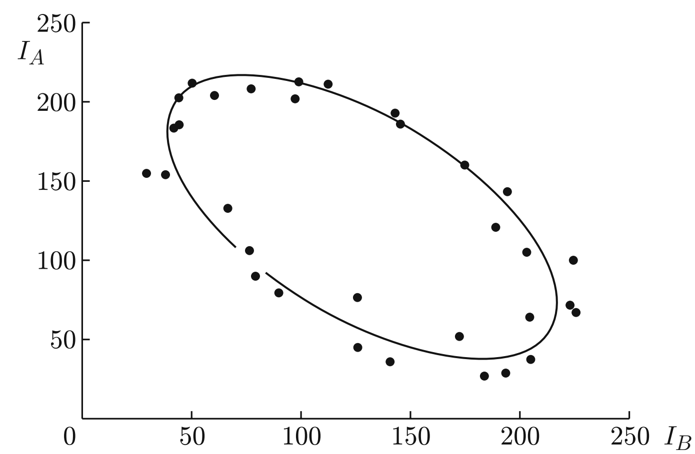

Fig. 2.

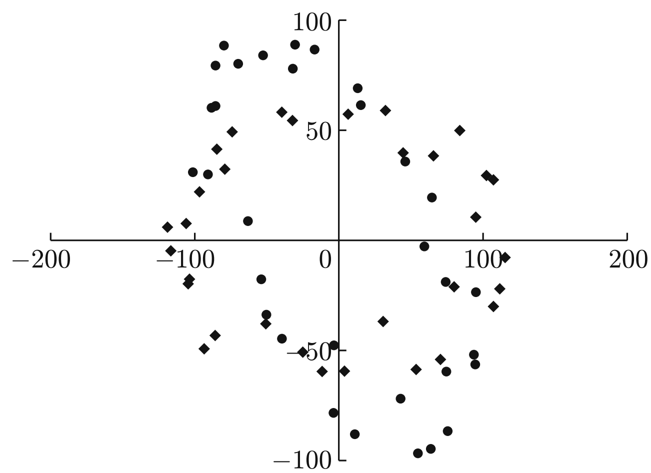

Fig. 3.

The mean intensity levels of the interference signals correspond to the coordinates of the center of the ellipse and are found as follows:

$$x0 = I_{01} = -\begin{vmatrix} a_{13} & a_{12} \\ a_{14} & a_{22} \end{vmatrix} \Big/ \begin{vmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{vmatrix}; \quad y0 = I_{02} = -\begin{vmatrix} a_{11} & a_{13} \\ a_{12} & a_{14} \end{vmatrix} \Big/ \begin{vmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{vmatrix} \tag{7}$$

The phase shift angles can be determined directly if the trajectory is made circular. To do this, we apply the following operations to the initial data vectors:

1) bring the center of the ellipse to the coordinate origin:

$$x1 = x - x0, \quad y1 = y - y0; \tag{8}$$

2) rotate the ellipse so that it is parallel to one of the coordinate axes:

$$\begin{bmatrix} x2 \\ y2 \end{bmatrix} = \begin{bmatrix} \cos\Omega & \sin\Omega \\ -\sin\Omega & \cos\Omega \end{bmatrix} \begin{bmatrix} x1 \\ y1 \end{bmatrix}; \tag{9}$$

the angle of rotation is given by the formula

$$\tan\Omega = \frac{2a_{12}}{a_{11} - a_{22}}; \tag{10}$$

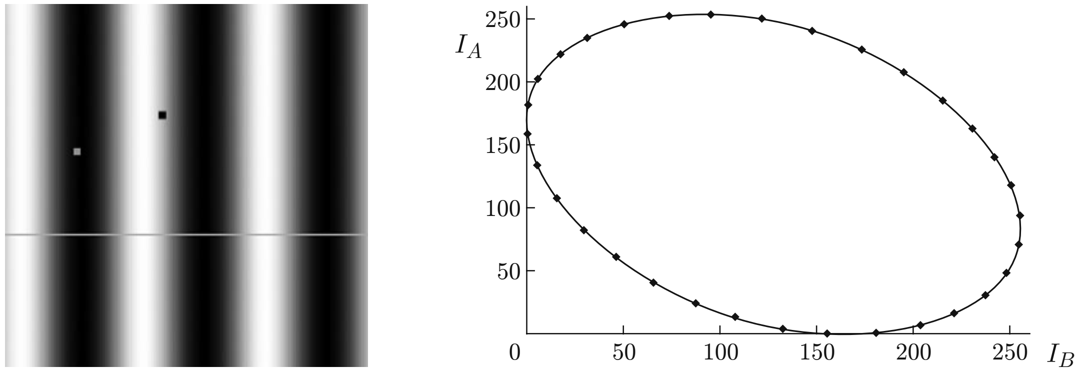

Fig. 4.

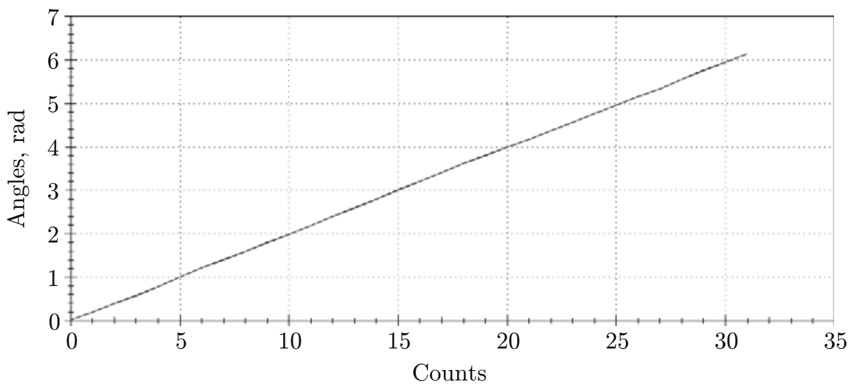

Fig. 5.

3) stretch the ellipse to a circle; the stretch coefficient γ is found from the canonical equation of the ellipse expressed in terms of its invariants

$$\lambda_0 x^2 + \lambda_1 y^2 + I_3/I_2 = 0, \tag{11}$$

where λ_0 and λ_1 are roots of the characteristic equation $\begin{vmatrix} a_{11}-\lambda & a_{12} \\ a_{21} & a_{22}-\lambda \end{vmatrix} = 0.$

From Eq. (11), it is seen that the ratio of the roots of the characteristic equation is equal to the ratio of the diameters of the ellipse:

$$\gamma = \sqrt{\lambda_0/\lambda_1}. \tag{12}$$

The stretching of the ellipse along the coordinate y is performed by the formula $y2 = y2/\gamma$.

Figures 3 shows the initial (points) and corrected (rhombi) trajectories of the interference signals. The phase angles are defined by the coordinates of the circular trajectory $\delta_i = \arctan(y2_i/x2_i)$.

An interference pattern with 256 intensity levels at two selected points and the trajectory constructed using 32 phase shifts are shown in Fig. 4. The phase shifts are arbitrary. Combined plots of the set and obtained phase shifts (32 shifts) are shown in Fig. 5. The standard deviation was 0.00412 rad, due to the discreteness of the intensity. The discreteness determines the error with which the phase angles of the shift can be obtained.

An interference pattern with a 10-percent error of intensity setting and the trajectory constructed using 32 phase shifts are shown in Fig. 6. Figure 7 shows plots of the real (curve *1*) and obtained (curve *2*) phase shifts with a 10-percent error of the intensity. In this case, the standard deviation of the obtained shifts from the set values was 0.0665 rad.

Thus, to improve the accuracy of finding shifts and, hence, the resulting phase differences, it is necessary to reduce the error in determining the field strength of the interferograms intensity shift.

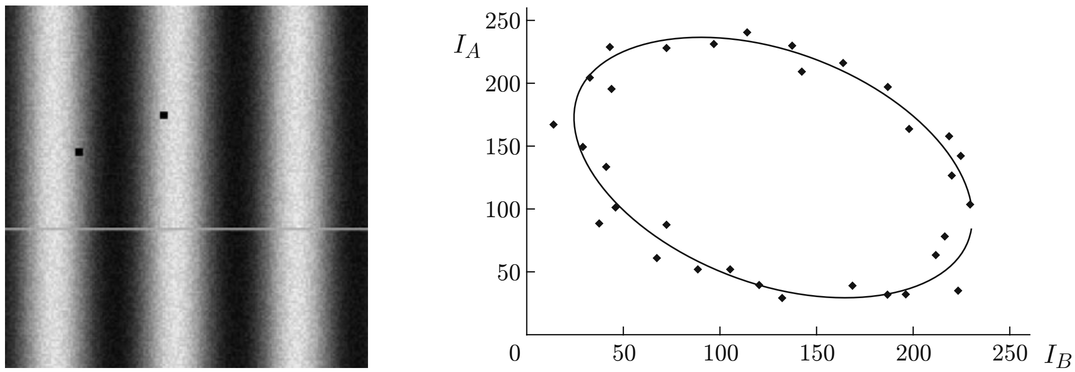

Fig. 6.

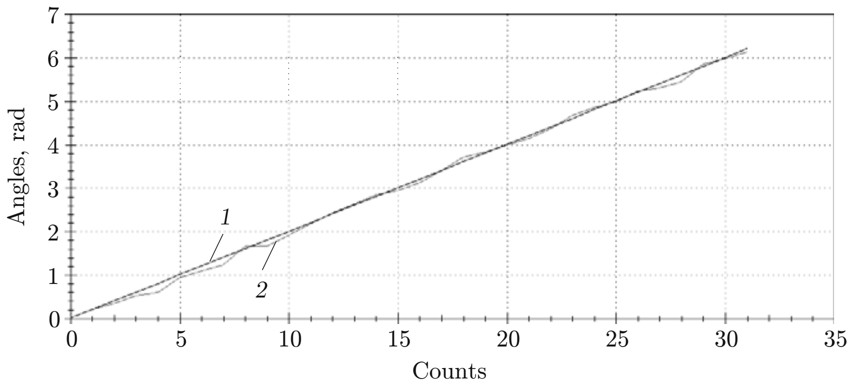

Fig. 7.

CONCLUSIONS

An efficient algorithm for decoding phase-shifting interference patterns was developed. In contrast to existing methods of phase measurements in interferometry, the proposed algorithm does not require a priori knowledge of the applied phase shifts and can therefore be used for random or uncontrollable shifts. The achieved accuracy of phase measurements is comparable to the measurement results obtained by atomic force microscopy and high-resolution electron microscopy [7]. For example, at a 5 percent error of measurement of the interferograms intensity field strength, the standard deviation of the measured phase does not exceed 0.01 rad, which corresponds to a resolution of 0.01 nm using a He–Ne laser with a wavelength of 633 nm.

This work was supported by the Russian Foundation for Basic Research (Grant No. 09-07-00133-a).

REFERENCES

1. K. Creath, "Phase-shifting speckle interferometry," Appl. Opt. **24** (18), 3053–3058 (1985).
2. J. Millerd, N. Brock, J. Hayes, et al., "Modern Approaches in Phase Measuring Metrology," Proc. SPIE. **5856**, 14–22 (2004).
3. P. Gao, B. Yao, N. Lindlein, et al., "Phase-Shift Extraction for Generalized Phase-Shifting Interferometry," Opt. Lett. **34** (22), 3553–3555 (2009).
4. J. E. Greivenkamp and J. H. Bruning, "Phase shifting interferometry," in *Optical Shop Testing*, Ed. by D. Malacara (Wiley, New York, 1992), Chapter 14, pp. 501–598.
5. J. Van Wingerden, H. J. Frankena, and C. Smorenburg, "Linear Approximation for Measurement Errors in Phase Shifting Interferometry," Appl. Opt. **30**, No. 19, 2718–2729 (1991).
6. G. A. Korn and T. M. Korn, *Mathematical Handbook for Scientists and Engineers* (Dover, New York, 2000).
7. L. I. Fedina, D. V. Sheglov, A. K. Gutakovskii, et al. "Precise Measurements of Nanostructure Parameters," Avtometriya **46** (4), 5–18 (2010) [Optoelectr., Instrum. Data Process. **46** (4), 301–311 (2010)].