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ANALYSIS AND SYNTHESIS OF SIGNALS AND IMAGES

Reconstruction of Images from a Series of Holograms Registered with a Low Resolution with the Help of a New Discretization Equation

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Abstract—A new method for image reconstruction from holograms with a high spatial resolution based on hologram shifts by values smaller than the size of aperture is considered. The method uses the discretization equation obtained with the help of generalized functions. Mathematical modeling of the process of reconstruction of a high-resolution image from low-resolution holograms is performed. The proposed method can be used to obtain digital holograms with the help of the photodetector array with a low resolution. This method does not require solving directly a large-dimensional system of algebraic equations.

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1. INTRODUCTION

To separate the real and imaginary images from the central beam of interferometer, one often uses the off-axis scheme of obtaining holograms proposed in [1–4] with angles between interfering beams of about 30° and larger. At that, the materials with a spatial resolution of 2000–4000 line pairs/mm are required. Nowadays, digital matrices for registering the intensity of digital images have a resolution of about 250 line pairs/mm. Therefore, to obtain the digital holograms with such a spatial resolution, it is necessary to reduce the angle between interfering beams. To register holograms, it is necessary to use optical schemes with small (smaller than 10°) angles [5]. However, at small interference angles, it is possible to record and reconstruct images only for the objects with a surface shape close to plane. On this account, when reconstructing images of 3D objects, it is necessary to increase the angle between interfering fronts, which requires increasing the spatial resolution when registering optical images. Therefore it is of importance to develop efficient methods for enhancing the spatial resolution when registering holograms. So, the attention of many researchers is payed to the problems of synthesis of a high-resolution image from a set of low-resolution rasters obtained by a subpixel shifting of the object image [6–9]. In radio engineering, such an approach is called synthesized aperture. In the scientific literature, such methods are often called the superresolution methods.

In [10], the method is considered to reconstruct an image from a digital hologram based on its spatial shifts by a value smaller than the aperture size and synthesis of a new hologram from the set of low-resolution holograms obtained as a result of spatial shifting. The method makes it possible to reconstruct the parts of the image corresponding to high harmonics (that are absent in the low-resolution raster). However, it has some disadvantages conditioned by the necessity of solving a large-dimensional system of algebraic equations [9].

The aim of this study is to describe a new method for reconstruction of images from holograms based on the discretization equation, which is obtained with the help of generalized functions and does not require solving directly a system of algebraic equations.

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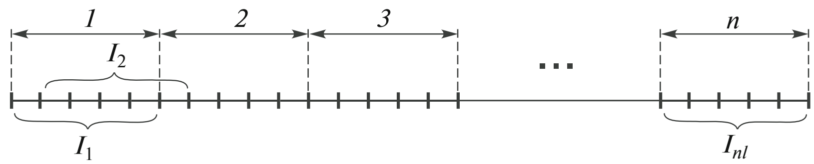

Fig. 1. Scheme of registering a low-resolution signal in the case of a subpixel shift over one row.

2. DISCRETIZATION OF FUNCTIONS WITH THE HELP OF A FINITE SET OF APERTURES AT A SUBPIXEL SHIFT

Real discretization is carried out by measuring a signal with the help of a set of detectors with some finite aperture (the area over which the averaging procedure is carried out). One uses apertures with various shapes (for example, elliptic, rhomboid, hexagonal); however, those are the apertures with rectangular and circular shapes that are used in most cases. In the case of averaging the values over the unit-aperture plane, is it convenient to describe the discretization procedure with the help of generalized functions [11–13].

Let us consider as an example, an aperture in the form of a rectangular function. We will determine the action of the normalized rectangular pulse $P_\tau(x) = \mathrm{rect}_\tau(x)$ on the function $f(x)$ describing the optical image intensity:

$$f_p(x) = \langle f(x), \mathrm{rect}_\tau(x)\rangle = \int_{-\infty}^{\infty} f(x)\,\mathrm{rect}_\tau(x)\,dx = \int_{x=k\,\Delta x-\tau/2}^{k\,\Delta x+\tau/2} f(x)\,dx. \tag{1}$$

Expression (1) describes averaging of the value of the function $f(x)$ at one element of aperture. The action of the shifted rectangular pulse $\mathrm{rect}_\tau\,(x - k\,\Delta x)$ on the function $f(x)$ can be represented as

$$f(k\,\Delta x) = \int_{-\infty}^{\infty} f(x)\,\mathrm{rect}_\tau\,(x - k\,\Delta x)\,dx = \int_{x=k\,\Delta x-\tau/2}^{k\,\Delta x+\tau/2} f(x)\,dx. \tag{2}$$

Expression (2) corresponds to averaging of the image at the kth aperture.

The general formulation of the problem of enhancing the spatial resolution in optical systems is determined in [14]. Figure 1 shows the scheme of registration of a one-dimensional signal when scanning it with an aperture with a width τ with a low resolution. Here, n is the number of elements of the raster with a low resolution, and nl is the number of high-resolution elements within the integrated aperture I_i, $i = 0, \ldots, n$.

As a result of discretization of an optical image by using an aperture whose size is nl elements with a unit step, we obtain n elements with a low resolution I_i.

For each image point, expression (2) is a discrete convolution

$$f(k) = \sum_{m=-\tau/2}^{\tau/2} f(k)\,\mathrm{rect}_\tau\,(k - m) = f(k) \otimes \mathrm{rect}_\tau(k). \tag{3}$$

In the Fourier region, the expression for discretization of a bounded function by using a finite set of apertures takes the following form [15]:

$$F_{\tau,\Delta x}(\omega) = ([F(\omega) \otimes \mathrm{sinc}(\omega N/2)]\Im\,(\mathrm{rect}_\tau(\omega)) \otimes \mathrm{comb}_{2\pi/\Delta x}(\omega), \tag{4}$$

where $([F(\omega) \otimes \mathrm{sinc}\,(\omega N/2)]\Im\,(\mathrm{rect}_\tau(\omega)))$ is the spectrum of the function obtained as a result of real discretization of the signal. It is possible to separate from this signal the part $[F(\omega) \otimes \mathrm{sinc}\,(\omega N/2)]$, which is the spectrum of the initial signal. It can be made by performing the element-wise normalization of the discretized function spectrum by the spectrum of aperture function $\Im\,(\mathrm{rect}_\tau(\omega))$. Performing the inverse Fourier transform, we obtain the function $f(k)$, where k is the index of the discrete array with a unit step (see Fig. 1). In [16], the algorithm is presented to find the high-resolution images from low-resolution images obtained by averaging over the set of apertures. Let us model the image reconstruction from holograms by using the proposed algorithm.

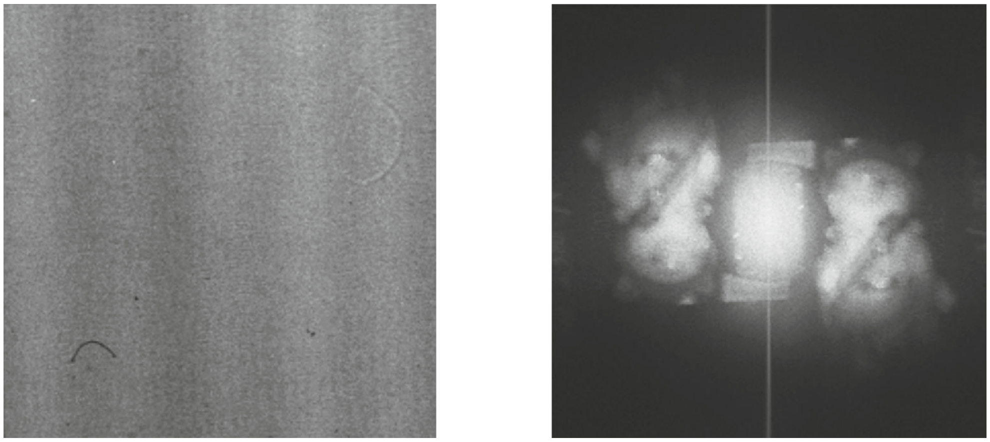

Fig. 2. Digitalized hologram (2048 × 2048 resolution cells) and the real (on the left) and imaginary (on the right) images reconstructed from it.

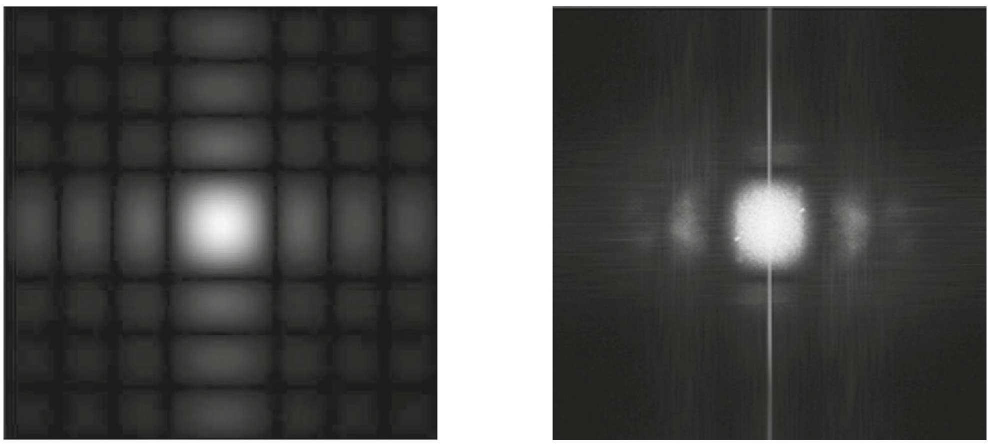

Fig. 3. Spectrum of the aperture function (the Fourier image of the rectangular aperture with 8 × 8 points (on the left)) and the image reconstructed from the smoothed hologram (on the right).

3. MODELING OF THE METHOD FOR ENHANCING THE RESOLUTION OF THE IMAGES RECONSTRUCTED FROM HOLOGRAMS AT A SUBPICEL SHIFT OF APERTURE

For modeling, real holograms obtained with the help of the off-axis scheme in [17] were used. The angle between interfering beams was 10°. The hologram was fixed at a PFG-03M photographic plate (the resolving power is not lower than 1570 line pairs/mm). To input the holograms into the computer, a stereoscopic modified microscope MBS-10 [18] was used. The microscope was modified by installing a photographic camera as an ocular and an automatized sample stage for moving the object. To digitalize the holograms, a microscope objective with an 8-fold magnification was used. When using such an objective, the spatial resolution was 1.5 μm. Then, a 3 × 3-mm fragment of the hologram was digitalized in the form of an eight-digit set of numbers. The size of the digitalized hologram was 2048 × 2048 resolution cells. Since the hologram is recorded in the Fraunhofer region, we used the Fourier transform to reconstruct the real and imaginary images. Figure 2 shows the digitalized hologram and the result of its reconstruction.

To model the discretization procedure with the help of the finite aperture, the hologram was averaged by using a floating window with a size of 8 × 8 resolution cells, which corresponds to 8 shifts by a unit step each with respect to x and 8 shifts by one point with respect to y. At that, 64 low-resolution holograms were obtained. Then, these holograms (with a size of 256 × 256 pixels) were unified to compose one hologram with a resolution of 2048 × 2048 resolution cells. Figure 3 shows the image reconstructed from the hologram averaged by means of a floating window (on the right). It is seen that a

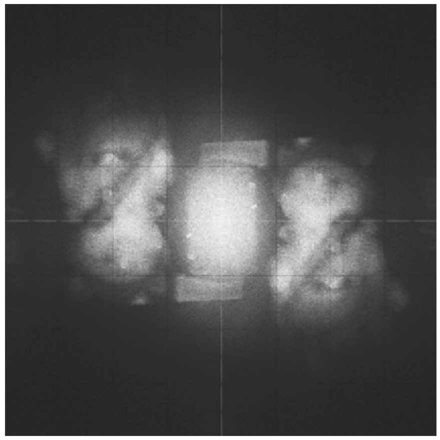

Fig. 4. Reconstructed image after element-wise division of the spectrum of the smoothed hologram by the spectrum of the aperture function.

reduction of the spatial resolution of the hologram after averaging has led to a loss of significant parts of the image. Figure 3 (on the left) shows the normalized amplitude spectrum of the aperture function (the interval of amplitude variations is from 0 to 1).

Figure 4 shows the reconstructed image after elementwise division of the amplitude spectrum of the hologram (see Fig. 3, on the right) by the amplitude spectrum of the aperture function (see Fig. 3, on the left). To eliminate the boundary effects in the process of image reconstruction, the digital hologram, before its smoothing, was completed with fields with a width identical to the aperture size). We can note that the reconstructed image in Fig. 4 coincides with the image reconstructed from the initial hologram with a high resolution (see. Fig. 2, on the right).

This example demonstrates the possibility of using the proposed algorithm for reconstruction of images from a series of holograms registered with a low discretization frequency by the synthesized aperture method.

4. CONCLUSIONS

In this study, the method for reconstruction of images from holograms with the help of synthesized aperture is considered. The method is based on the completion of the initial hologram with the results of measurements obtained when performing its spatial shift by a value smaller than that of the aperture of the element of the photodetector array. To reconstruct the images, it is necessary to obtain the hologram by using the procedure described in [17], then, divide the elements of the reconstructed images by the spectrum of the aperture function. It is shown that, when the aperture function is a priori known, even in the case of a low resolution of the initial holograms, it is possible to reconstruct the image with a high resolution. The results of modeling of this algorithm are presented. This method, as distinct from that described in [10], does not require direct solving a large-dimensional system of algebraic equations.

CONFLICT OF INTEREST

The authors declare that they have no conflicts of interest.

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