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Method for increasing the dynamic range of digital images based on modular arithmetic

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Abstract. The article proposes a new method of synthesis of digital images from several digital images to increase the dynamic range of intensity. The method is based on modular arithmetic. The increased range is divided into several residual classes. The full value of the intensity is calculated as the solution of the system of comparisons of residues for given modules. An analysis is made of the stability of the proposed method to the nonlinearity of the transfer characteristic and digital image recording devices under various types of interference. To this end, the maximum range for changing the brightness of a digital image is limited. By selecting the magnitude of the modules and the maximum brightness value, the distance between the diagonals in the decision table is selected so that this distance is greater than the level of distortion and (or) interference. Experimental validation of the method has been performed.

1. Introduction

Methods of non-contact optical measurements based on structured lighting use the projection of known paintings and the registration of digital images distorted depending on the profile of the object [1-4]. The accuracy of measurements of the phase methods of structured lighting is largely determined by the error of the recorded images, which depends on the dynamic range of the brightness field of the digital image. Modern optical projection devices have a resolution of 8 bits, and registration devices have a resolution of 8-14 bits. However, to perform high-precision measurements, it is necessary to increase the dynamic range during projection and registration. An increase in bit depth is possible using modular arithmetic [5-7]. In optical interferometry, this approach is used for solving the problem of phase uncertainty [8, 9]. In this paper, we propose a new method that allows increasing the range of changes in the intensity of digital images with the same hardware capabilities of their projection and registration devices.

2. Basic of modular arithmetic

The method of modular arithmetic consists of operating not directly with the number X , but with its residues from dividing by some numbers m_i

$$b_1 = X \bmod m_1 \quad (1a)$$

$$b_2 = X \bmod m_2 \quad (1b)$$

The initial data are the results of measurements of b_1 and b_2 of the same value X for different values of the periods m_1 and m_2 .

The maximum range of unambiguous determination of absolute values is determined by the largest, mutually simple factors in the values of the periods. If the modules are mutually simple, then the maximum range is equal to their product.

Let each integer a and b correspond to a certain remainder r from dividing by a positive integer m , which is called the module. If two integers correspond to the same remainder, then they are called equidistant modulo m . The comparability of two numbers is written as

$$a \equiv b \pmod{m} \quad (2)$$

where the sign (\equiv) denotes a comparison operation. In this case, the system of equations (2) can be written as a system of comparisons

$$X \equiv b_1 \pmod{m_1}, \quad X \equiv b_2 \pmod{m_2} \quad (3)$$

If the modules are coprime numbers, then in a certain range there is a unique solution. To find the number from the set of residues, one can use the “Chinese” remainder theorem [10].

Let the numbers M_s and N_s be determined from the conditions.

$$m_1 \cdot m_2 = M_s m_s, \quad (4)$$

$$M_s N_s \equiv 1 \pmod{m_s} \quad (5)$$

Then the set of values of X satisfying the comparison system (6) is determined by comparison and let.

$$X_0 = M_1 N_1 b_1 + M_2 N_2 b_2, \quad (6)$$

$$X \equiv X_0 \pmod{m_1 \cdot m_2} \quad (7)$$

If the values measured within the period are postponed vertically and horizontally, it is possible to get a table whose values will satisfy the absolute values of the desired measured value. Figure 1 shows the table of solutions of the comparison system with two relatively simple modules: $m_1 = 11$ and $m_2 = 17$.

It can be seen that the numbers increase from 0 to $m_1 - 1$ sequentially along the main diagonal, and then along the diagonals shown by the arrows in Figure 1. Unfortunately, it is difficult to trace the full turns of the two-dimensional table. Only the initial turn is completely placed on one diagonal, other turns begin on the zero lines and continue on the diagonals starting with the corresponding values on the columns. Therefore, it is possible to store two arrays of the number of turns in rows and columns (in Figure 1 – the second row and second column contain information about the number of turns).

This process can be simplified. For this, it is necessary to store the expanded array $n[i]$ of size $m_1 + m_2 - 1$, where i varies from m_1 to $m_2 - 1$ (Figure 2). Negative indices in this array will correspond to the second column in Figure 1.

b1	b2	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
0	0	14	11	8	5	2	16	13	10	7	4	1	15	12	9	6	3	
1	3	0	154	121	88	55	22	176	143	110	77	44	11	165	132	99	66	33
2	6	34	1	155	122	89	56	23	177	144	111	78	45	12	166	133	100	67
3	9	68	35	2	156	123	90	57	24	178	145	112	79	46	13	167	134	101
4	12	102	69	36	3	157	124	91	58	25	179	146	113	80	47	14	168	135
5	15	136	103	70	37	4	158	125	92	59	26	180	147	114	81	48	15	169
6	1	170	137	104	71	38	5	159	126	93	60	27	181	148	115	82	49	16
7	4	17	171	138	105	72	39	6	160	127	94	61	28	182	149	116	83	50
8	7	51	18	172	139	106	73	40	7	161	128	95	62	29	183	150	117	84
9	10	85	52	19	173	140	107	74	41	8	162	129	96	63	30	184	151	118
10	13	119	86	53	20	174	141	108	75	42	9	163	130	97	64	31	185	152
11	16	153	120	87	54	21	175	142	109	76	43	10	164	131	98	65	32	186

↑
Number of turns

← Number of turns

Figure 1. The sequential change in the numbers in the decision table of the system of comparisons with $m_1 = 11$ and $m_2 = 17$ (b_1 varies from 0 to 10, b_2 from 0 to 16).

Then

$$X = n[b_2 - b_1] \cdot m_1 + b_1 \quad (8)$$

for example: if $b_1 = 10$ and $b_2 = 0$: $b_2 - b_1 = -10$, $n[b_2 - b_1] = 13$, $n[b_2 - b_1] \cdot m_1 = 143$, $n[b_2 - b_1] \cdot m_1 + b_1 = 153$ (Figure 2).

b1	b2	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16									
13	10	7	4	1	15	12	9	6	3	0	14	11	8	5	2	16	13	10									
											0	154	121	88	55	22	176	143	110								
											34	1	155	122	89	56	23	177	144	111							
											68	35	2	156	123	90	57	24	178	145	112						
											102	69	36	3	157	124	91	58	25	179	146	113					
											136	103	70	37	4	158	125	92	59	26	180	147	114				
											170	137	104	71	38	5	159	126	93	60	27	181	148	115			
											17	171	138	105	72	39	6	160	127	94	61	28	182	149	116		
											51	18	172	139	106	73	40	7	161	128	95	62	29	183	150	117	
											85	52	19	173	140	107	74	41	8	162	129	96	63	30	184	151	118
											119	86	53	20	174	141	108	75	42	9	163	130	97	64	31	185	152
											153	120	87	54	21	175	142	109	76	43	10	164	131	98	65	32	186

Figure 2. The positions of the brightness values in the decision table (Fig. 1)

However, it can be seen from the tables in Figures 1 and 2 that even small errors in the initial data lead to a reading falling onto adjacent diagonals and this leads to significant errors in determining the result. Therefore, the method is significantly unstable to the effects of destabilizing factors. For increasing its stability, it is necessary to develop a method for correcting erroneous data.

3. Correction of comparisons in the presence of erroneous initial data

If limit the maximum brightness range is limited, a gap is formed between the diagonals (Figure 1). These diagonals will contain numbers falling into the selected range, and the values between the diagonals will lie outside this range (Figure 3).

If there is a priori information about the maximum range of brightness deviations due to the nonlinearity of the transfer characteristic of the input device, additive or multiplicative noise, then by restricting these values to the allowable range the correct measurements give the exact value of the brightness value. From the erroneous data, we get numbers that are not in the range of acceptable values. These numbers go beyond the selected range.

In Figure 3, the region (region of gross misses) is shaded over, into which the measured value may fall in the presence of measurement error. It is necessary to correct these erroneous values in some way. There are two ways to do this:

- ignore erroneous values;
- correct them to the nearest correct diagonal.

Working with an extended array $n[i]$ makes it easy to organize work in the presence of failed (erroneous) initial data. For this, it is necessary to establish a priori what maximum range the measured object has. Let, for example, be five turns. The maximum range is equal $5m_1 + 52 = 317$. In this case, we can specify an array in which all values greater than 5 will be zero. Moreover, all diagonals, tables starting from zero, will also be equal to zero. Thus, only the values of the points falling on the diagonals will be calculated (Figure 3). Correction of bad points can be performed as follows. We fill in the closest value of the allowable diagonal to the zero gaps in the array (Figure 4).

						b2	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5	0	0	0	0	0	0	4	0	0	0	
1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5	5	5	5	5	5	5	5	4	4	4	4

(a)

(b)

Figure 3. Fragment of an array with modules: $m_1 = 53$ and $m_2 = 63$. Diagonals (a) less than or equal to 5 and (b) supplemented with acceptable diagonal values are shown in yellow.

Then the values of the numbers in the comparison table can be calculated as

$$X = n[b_2 - b_1] \cdot m_1 + b_1 \quad (9)$$

Figure 5 shows a solution table for mutually simple modules $m_1 = 167$ and $m_2 = 211$.

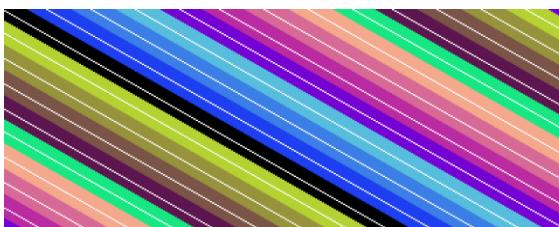


Figure 4. Comparison table with modules $m_1 = 167$ and $m_2 = 211$. The first 13 diagonals are selected.

The figure shows the areas in which values (b_1, b_2) are adjusted to the nearest diagonal. These areas are highlighted in different colours; valid diagonals are indicated in white. Since they $m_1 = 167$ and $m_2 = 211$ are coprime numbers, the maximum measurement range m_{\max} can be 211 periods.

4. Experiments and Results

For experimental verification of the algorithm, two brightness wedges were projected onto the screen. The experimental setup consists of a 4K projector VPL-VW260ES, which has a resolution of 4096 by 2160 pixels. For register digital images, a CANON EOS M50 camera connected to a computer is used. The Canon EOS M50 has a CMOS-type image sensor with a resolution of 24.1 MP and a physical frame size of 22.3 x 14.9 mm (APS-C format). The maximum format of the digitized frame is 6000x4000. Thanks to the new DIGIC 8 processor, support for video recording in 4K format is provided. The maximum video resolution is 3840x2160 pixels at 25 frames per second. Figure 5 shows a general view of the installation.



Figure 5. General view of the installation.

Figure 6 shows the registered brightness fields of two digital images with a resolution of 8 bits.

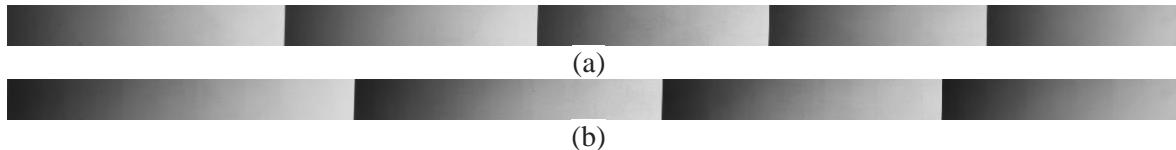


Figure 6. Fields of the brightness of the digital image by modules: (a) - 167 and (b) - 211.

For achieve algorithm stability, the maximum range must be reduced. In this case, the distance between the diagonals becomes more significant than the amplitude of the noise. If we limit the maximum brightness range to 13 bands, then the maximum brightness value will be equal to 2743, which is more than ten times the maximum brightness value for an eight-bit digital image. Figure 7 shows a comparison table for this case.

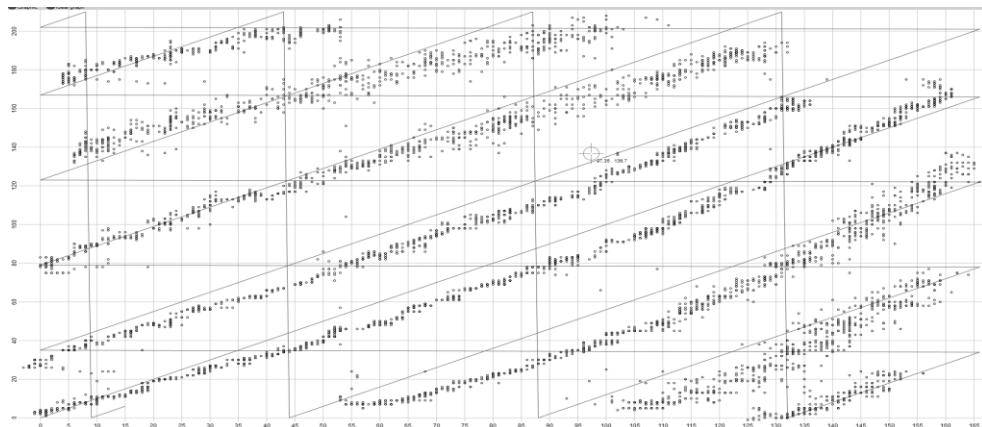


Figure 87. Comparison table.

Error brightness values are corrected by projecting them onto the nearest allowed diagonal. Figure 8 shows the results of the synthesis of digital images with correction according to the proposed method for two values $m_{\max} = 21m_1$ - (a) and $m_{\max} = 13m_1$ - (b).

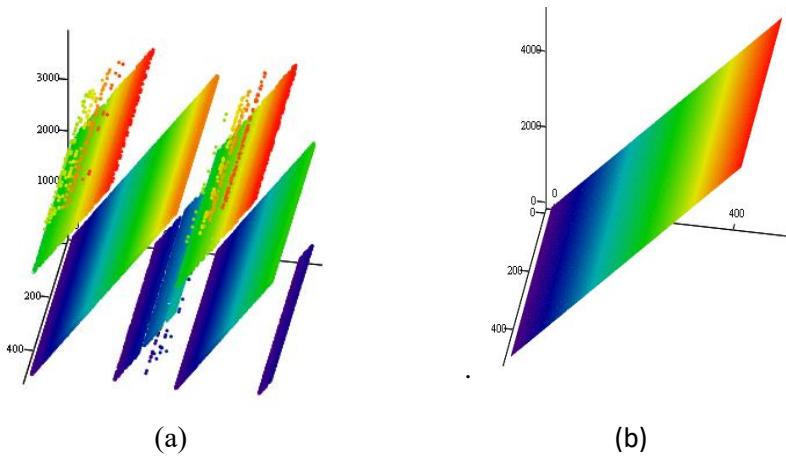


Figure 8. Digital image reconstruction results while limiting the range: (a) $m_{\max} = 21m_l$ and (b) $m_{\max} = 13m_l$.

Thus, from two 8-bit digital images, an image with possible gradations of grey is obtained, which corresponds to a 15-bit representation of a digital image. If it needs to increase the brightness range, then it is possible using not two but three or more mutually simple modules, for example, with three mutually simple modules, for example, with three mutually simple modules we get $m_1m_2m_3 = 211 \cdot 167 \cdot 113 = 3981781$ that corresponds to a 20-bit representation of a digital image.

5. Conclusions

The article discusses a new method for synthesizing a digital image with an increased range of brightness from a set of digital images with low brightness. A method is proposed for increasing the stability of the method by limiting the maximum brightness of the synthesized digital image. For the first time, a practical implementation of an algorithm that is resistant to noise and distortion of the brightness field has been performed.

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