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Image Reconstruction from Digital Holograms Obtained by Specifying Random Phase Shifts

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Abstract—A new method for image reconstruction from a series of digital holograms obtained by stepped phase shift with random values of phase shifts is under consideration. It is shown that images reconstructed from a digital hologram in a Fresnel region at a distance matching a distance from the object to the hologram consist of two parts, one of which is a clear image of the object and the other one being formed by incorrect phase shifts consists of a set of defocused images of the object. At the same time, the quality of the reconstructed image is slightly reduced. This effect eliminates the necessity to use precision systems for setting the phase shift and significantly reduces requirements for stabilizing the optical device.

Keywords: digital holography, step phase shift, interferometry, random phase shifts.

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INTRODUCTION

Digital holography significantly increases the productivity and practicality of interference measurement methods. Modern digital holography methods are based on recording an interference pattern intensity using matrix photodetectors. In [1], the digital image reconstruction method was proposed for the first time. Digital holography was further developed after the possibility of directly recording holograms in digital form [2–4]. A large contribution to this field of research was made by Russian scientists [5–8].

The real and imaginary images are often separated from the central beam using the off-axis hologram production scheme proposed in [9], with the angles between interfering beams of the order of 30° and larger. This requires materials with a spatial resolution of 2000–4000 lines/mm. Digital matrices used to record intensity currently have a resolution of fewer than 250 lines/mm, so obtaining holograms requires that the angle between the interfering beams is reduced [10]. At small angles, spectra inevitably overlap in different diffraction orders

The use of stepped phase shift methods for obtaining holograms prevents problems associated with the elimination of a zero-order diffraction and a double image [8, 11–13].

The aim of this work is computer simulation and experimental study of the self-focusing effect of images reconstructed from holograms obtained by the stepped phase shift method.

FORMATION OF A MATHEMATICAL HOLOGRAM

In contrast to the formation of classical holograms containing information only about intensity, the stepped phase shift can be used to form a mathematical hologram [7]. In contrast to digital holograms, which are intensity patterns, a mathematical hologram is the complex field

$$G(x, y) = a_p(x, y)\, \mathrm{e}^{-i\varphi_p(x,y)}, \tag{1}$$

where $a_p(x, y)$ denotes the field amplitude and $\varphi_p(x, y)$ is the phase of the field scattered from the object in the hologram plane.

In order to determine the amplitude and phase of a mathematical hologram, a series of holograms is formed by introducing different phase shifts into a reference beam [14, 15]. A generalized scheme of the algorithm for determining the phase difference for a various number of random phase shifts is described in [16–20]. The phase difference $\Delta\varphi = \varphi_p(x, y) - \varphi_r(x, y)$ between the object beam $\varphi_p(x, y)$ and the reference beam $\varphi_r(x, y)$ can be determined as

$$\Delta\varphi = \arctan\Big(\frac{\mathbf{I}\cdot\mathbf{C}^{\perp}}{\mathbf{I}\cdot\mathbf{S}^{\perp}}\Big), \tag{2}$$

where $\mathbf{I} = (I_0, I_1, \ldots, I_{m-1})$ is the intensity vector of set m of holograms obtained with the phase shifts $\delta_0, \delta_1, \ldots, \delta_{m-1}$; $\mathbf{C} = (\cos\delta_0, \ldots, \cos\delta_{m-1})^{\top}$; $\mathbf{S} = (\sin\delta_0, \ldots, \sin\delta_{m-1})^{\top}$; $\mathbf{I}^{\perp}$ is the vector orthogonal to the vector $\mathbf{I}$, $(\mathbf{I}\cdot\mathbf{I}^{\perp}) = 0$.

Knowing the phase difference $\Delta\varphi$ and the reference wave phase $\varphi_r(x, y)$, one can determine the original phase distribution for the object wave phase $\varphi_p(x, y)$.

The formation of the mathematical hologram (1) requires determining the amplitude of the initial wave $a_p(x, y)$, which is easily carried out provided that a plane wave with a constant amplitude is used as a reference beam. In this case, it suffices to calculate the value $B(x, y) = 2a_p(x, y)a_r(x, y)$, where $a_r(x, y)$ is the amplitude of the reference beam

$$B = \frac{1}{|\mathbf{C}\cdot\mathbf{S}^{\perp}|}\sqrt{(\mathbf{I}\cdot\mathbf{S}^{\perp})^2 + (\mathbf{I}\cdot\mathbf{C}^{\perp})^2}\,. \tag{3}$$

An object image is obtained by making a discrete Fresnel transform over a mathematical hologram [21]

$$\Gamma(r, s) = -\exp\Big\{i\,\frac{\pi}{\lambda d}\Big[\Big(\Delta\xi\Big(r - \frac{N_x}{2}\Big)\Big)^2 + \Big(\Delta\eta\Big(s - \frac{N_y}{2}\Big)\Big)^2\Big]\Big\} \times$$

$$\times\,\Im\Big(G(k, l)\exp\Big\{i\,\frac{\pi}{\lambda d}\Big[\Big(\Delta\xi\Big(k - \frac{N_x}{2}\Big)\Big)^2 + \Big(\Delta\eta\Big(l - \frac{N_y}{2}\Big)\Big)^2\Big]\Big\}\Big), \tag{4}$$

where λ is the wavelength of the coherent source that is used for illumination, d is the distance containing the complex distribution of the wave field scattered from the hologram recording plane, and $G(k, l)$ denotes the counts of the mathematical hologram (1) with sampling intervals $\Delta\xi$ and $\Delta\eta$; N_x and N_y are the numbers of the counts.

If d matches the distance from the object to the hologram, then the image is focused.

The main disadvantage of the stepped phase shift method is the need to know the exact values introduced into the reference beam of phase shifts. The errors in setting phase shifts or using incorrect values of phase shifts in a decoding equation lead to errors in measuring the phase and amplitude of the image of the original object [16]. Therefore, in optical circuits, precision devices are used for introducing shifts and schemes for stabilizing the device from external vibrations (holographic tables).

It can be shown that image reconstruction based on mathematical holograms causes errors that negligibly lower the image quality.

INFLUENCE OF INACCURATELY SETTING PHASE SHIFTS ON THE FORMATION OF MATHEMATICAL HOLOGRAMS

The Euler equation $a\exp^{-i\varphi} = a(\cos\varphi - i\sin\varphi)$ and the mathematica hologram (1) can be written in the form

$$G(x, y) = a_p(x, y)\cos(\varphi_p(x, y)) - ia_p(x, y)\sin(\varphi_p(x, y)). \tag{5}$$

In the phase difference equation (2), $\cos(\varphi_p(x, y)) \approx \mathbf{I}^{\perp}\cdot\mathbf{S}$ and $\sin(\varphi_p(x, y)) \approx \mathbf{I}^{\perp}\cdot\mathbf{C}$. Therefore, Eq. (5) has the form

$$G(x, y) = a_p(x, y)\mathbf{I}^{\perp}\cdot\mathbf{S} - ia_p(x, y)\mathbf{I}^{\perp}\cdot\mathbf{C}. \tag{6}$$

Let the value of the phase sine at four shifts be represented as

$$\cos(\Delta\varphi) = \mathbf{I}^{\perp} \cdot \mathbf{C} = \left[M \begin{pmatrix} I_0 \\ I_1 \\ I_2 \\ I_3 \end{pmatrix} \right] \begin{pmatrix} \cos(\delta_0 + \Delta\delta_0) \\ \cos(\delta_1 + \Delta\delta_1) \\ \cos(\delta_2 + \Delta\delta_2) \\ \cos(\delta_3 + \Delta\delta_3) \end{pmatrix}, \tag{7}$$

where the matrix M has the same form as in Eq. (6), and $\Delta\delta_i$ is the error in determining the phase shift. Next, the sine is written in the form of a sum of the correct value and some error:

$$\cos(\delta_i + \Delta\delta_i) = \cos(\delta_i) + \cos(\delta_i + \Delta\delta_i) - \cos(\delta_i) = C + \Delta C. \tag{8}$$

Then the expression for the difference of the two cosines $\cos(\alpha) - \cos(\beta) = -2\sin((\alpha + \beta)/2) \cdot \sin((\alpha - \beta)/2)$ is used to obtain an expression for an error formed by incorrectly setting the phase shift angles:

$$\Delta C = \cos(\delta_i + \Delta\delta_i) - \cos(\delta_i) = -2\sin(\delta_i + \Delta\delta_i/2) \cdot \sin(\Delta\delta_i/2). \tag{9}$$

The expression $\sin(\alpha + \beta) = \sin\alpha \cdot \cos\beta + \cos\alpha \cdot \sin\beta$ is used to transform Eq. (9) and obtain

$$\Delta C = -2\sin(\delta_i + \Delta\delta_i/2) \cdot \sin(\Delta\delta_i/2)$$

$$= -2\sin\delta_i \cdot \cos(\Delta\delta_i/2) \cdot \sin(\Delta\delta_i/2) - 2\cos\delta_i \cdot \sin(\Delta\delta_i/2) \cdot \sin(\Delta\delta_i/2). \tag{10}$$

With account for the known trigonometric equation $\sin\alpha \cdot \cos\beta = [\sin(\alpha - \beta) + \sin(\alpha + \beta)]/2$, we have

$$\Delta C = -\sin\delta_i \cdot \sin\Delta\delta - 2\cos\delta_i \cdot \sin(\Delta\delta_i/2) \cdot \sin(\Delta\delta_i/2)$$

$$= -\sin\delta_i \cdot \sin\Delta\delta - 2\cos\delta_i \cdot [\sin(\Delta\delta_i/2)]^2. \tag{11}$$

Similarly, we obtain the dependence of the error in determining the phase sine from incorrectly setting the phase shifts:

$$\sin(\Delta\varphi) = \mathbf{I}^{\perp} \cdot \mathbf{S} = \left[M \begin{pmatrix} I_0 \\ I_1 \\ I_2 \\ I_3 \end{pmatrix} \right] \begin{pmatrix} \sin(\delta_0 + \Delta\delta_0) \\ \sin(\delta_1 + \Delta\delta_1) \\ \sin(\delta_2 + \Delta\delta_2) \\ \sin(\delta_3 + \Delta\delta_3) \end{pmatrix}, \tag{12}$$

$$\sin(\delta_i + \Delta\delta_i) = \sin(\delta_i) + \sin(\delta_i + \Delta\delta_i) - \sin(\delta_i) = S + \Delta S, \tag{13}$$

$$\Delta S = \sin(\delta_i + \Delta\delta_i) - \sin(\delta_i) = 2\cos(\delta_i + \Delta\delta_i/2) \cdot \sin(\Delta\delta_i/2) =$$

$$= 2\cos\delta_i \cdot \cos(\Delta\delta_i/2) \cdot \sin(\Delta\delta_i/2) - 2\sin\delta_i \cdot [\sin(\Delta\delta_i/2)]^2 =$$

$$= \cos\delta_i \cdot \sin\Delta\delta_i - 2\sin\delta_i \cdot [\sin(\Delta\delta_i/2)]^2. \tag{14}$$

The amplitude of the object beam in the hologram field can be calculated from expression (3). In the case of a plane reference beam, with an accuracy to factor $|\mathbf{S} \cdot \mathbf{C}^{\perp}|^{-1}$

$$a_p \cong \sqrt{(\mathbf{I} \cdot \mathbf{S}^{\perp})^2 + (\mathbf{I} \cdot \mathbf{C}^{\perp})^2}. \tag{15}$$

The factor $|\mathbf{S} \cdot \mathbf{C}^{\perp}|^{-1}$ determines the average brightness of the hologram, and it changes in the cases of incorrect calculating or setting phase shifts.

Thus, in the case of random phase shifts, the mathematic hologram can be represented as a superposition of a correct hologram obtained at really introduced shifts and a hologram matching some error in setting phase shifts

$$G_{error}(x,y) = G(x,y) + G_1(x,y), \tag{16}$$

where the expression $G_1(x,y)$ for four shifts takes the form

$$G_1(x,y) \cong \left[M\begin{pmatrix} I_0 \\ I_1 \\ I_2 \\ I_3 \end{pmatrix}\right] \cdot \begin{pmatrix} \cos(\delta_0)\cdot\sin(\Delta\delta_0) \\ \cos(\delta_1)\cdot\sin(\Delta\delta_1) \\ \cos(\delta_2)\cdot\sin(\Delta\delta_2) \\ \cos(\delta_3)\cdot\sin(\Delta\delta_3) \end{pmatrix} - i\left[M\begin{pmatrix} I_0 \\ I_1 \\ I_2 \\ I_3 \end{pmatrix}\right] \cdot \begin{pmatrix} \sin(\delta_0)\cdot\sin(\Delta\delta_0) \\ \sin(\delta_1)\cdot\sin(\Delta\delta_1) \\ \sin(\delta_2)\cdot\sin(\Delta\delta_2) \\ \sin(\delta_3)\cdot\sin(\Delta\delta_3) \end{pmatrix} -$$

$$- 2\left[M\begin{pmatrix} I_0 \\ I_1 \\ I_2 \\ I_3 \end{pmatrix}\right] \cdot \begin{pmatrix} \sin(\delta_0)\cdot[\sin(\Delta\delta_0/2)]^2 \\ \sin(\delta_1)\cdot[\sin(\Delta\delta_1/2)]^2 \\ \sin(\delta_2)\cdot[\sin(\Delta\delta_2/2)]^2 \\ \sin(\delta_3)\cdot[\sin(\Delta\delta_3/2)]^2 \end{pmatrix} +$$

$$+ 2i\left[M\begin{pmatrix} I_0 \\ I_1 \\ I_2 \\ I_3 \end{pmatrix}\right] \cdot \begin{pmatrix} \cos(\delta_0)\cdot[\sin(\Delta\delta_0/2)]^2 \\ \cos(\delta_1)\cdot[\sin(\Delta\delta_1/2)]^2 \\ \cos(\delta_2)\cdot[\sin(\Delta\delta_2/2)]^2 \\ \cos(\delta_3)\cdot[\sin(\Delta\delta_3/2)]^2 \end{pmatrix}. \tag{17}$$

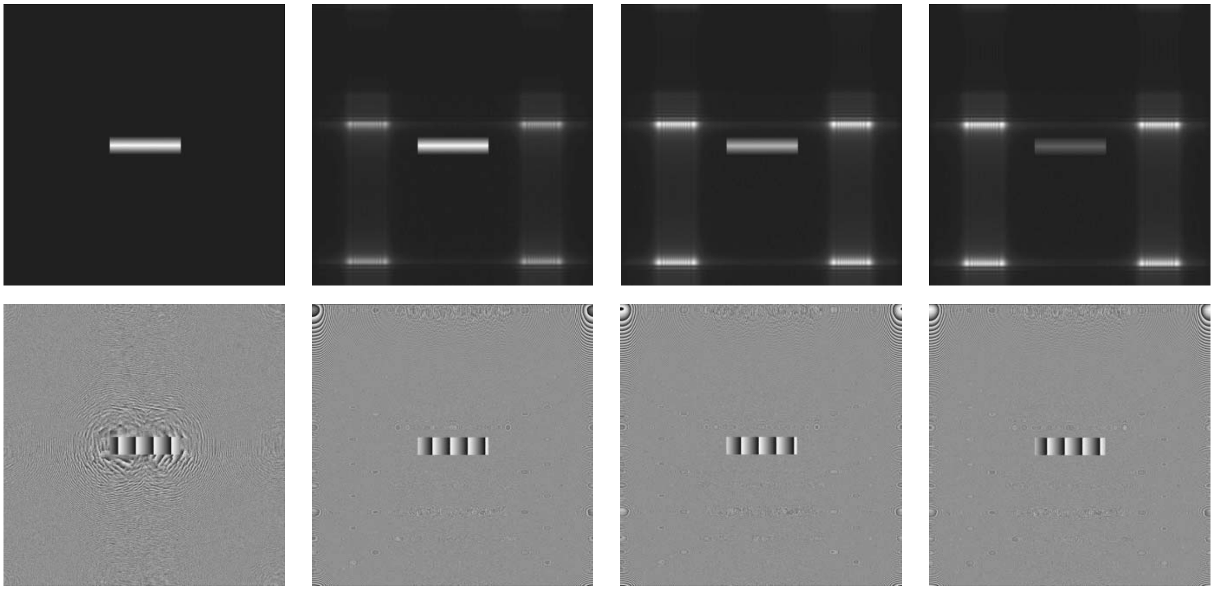
In order to reconstruct the amplitude and phase of an object beam, it is necessary to make the Fresnel transform over a mathematical hologram. It turns out that, in this case, the object image is formed from a hologram with the correct shifts of $G(x,y)$ and from the erroneous part of the hologram $G_1(x,y)$ (16). The first image is focused clearly, and the second is blurry. To demonstrate this phenomenon, we carry out a computer simulation of the amplitude and phase reconstruction for arbitrary phase shift values.

COMPUTER SIMULATION OF AMPLITUDE AND PHASE RECONSTRUCTION FOR ARBITRARY PHASE SHIFT VALUES

Let a series of holograms from an object with amplitude and phase shown in Fig. 1 be obtained. The phase of the object is distributed in the range from $-\pi$ to π. The process of computer simulation of holograms is given in [21, 22]. To form holograms, a reference front with some known phase shifts is added to the wavefront scattered from the object. It is assumed that the exact value of these shifts is unknown and the reconstruction is carried out at some other arbitrarily chosen values.

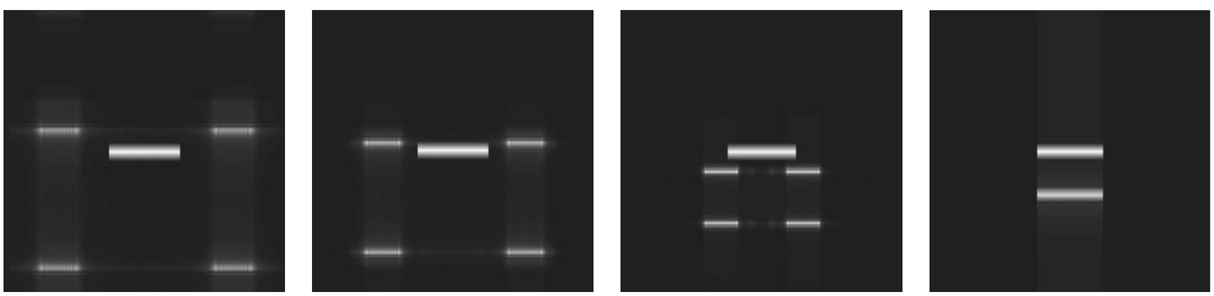
Figure 2 shows a series of holograms obtained with shifts between the reference and object beams of 0, 90, 180, and 270°. We determine the mathematical hologram at some random shifts between the reference and object beams and reconstruct the amplitude and phase in the observation plane, which is at the same distance as the plane where the holograms from the object are obtained. The bottom row in the figure shows the phase shift values at which decoding is performed.

Fig. 1. Distribution of complex amplitudes in the plane tangential to the object: the amplitude is on the left, and the phase is on the right.



0, 90, 180, 270°   0, 10, 20, 30°   90, 10, 20, 180°   90, 0, 30, 270°

Fig. 2. Distribution of the amplitudes and phases reconstructed from digital holograms in an observation plane with incorrectly set shift angles.



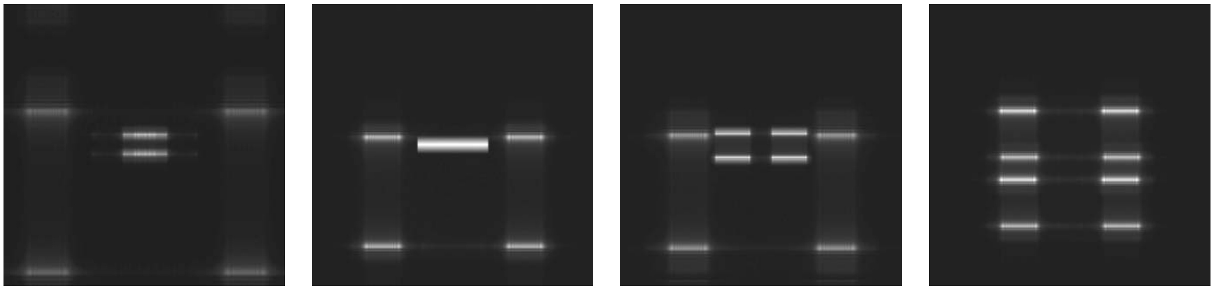
135 mm   200 mm   500 mm   In the Fraunhofer region

Fig. 3. Amplitude distribution in the observation plane at different distances from the hologram to the observation plane.

It can be seen from the figure that, despite obvious errors in setting shifts during decoding, the amplitude and phase of the object beam correspond to the object.

Aside from the object image, we encounter some distortions caused by incorrect shifts when determining the amplitude. These distortions depend on the distances at which the hologram is recorded and the image is reconstructed. With an increase in the distortion distance, an image is formed corresponding to a double image of the object amplitude in the Fraunhofer region. This property can be demonstrated by forming a mathematical hologram with unknown phase shifts (Fig. 3). When an image is reconstructed from a mathematical hologram, the shift angles of 0, 10, 20, 30° are selected arbitrarily.

The effects of image focusing and distortion defocusing, caused by incorrectly setting the phase shift angles can be shown by varying the distance at which the amplitude and phase are reconstructed using the mathematical hologram (Fig. 4). Let the plane in which the holograms are recorded be located at a distance of 135 mm from the object. We consider the reconstruction of holograms at various distances to the reconstruction plane. It can be seen that the image in the plane located at a distance of 135 mm from the hologram is focused, but the distortions are not focused in the double image.



100 mm | 135 mm | 200 mm | In the Fraunhofer region

Fig. 4. Amplitude distribution after reconstruction, with the shift angles being set at a different distance from the object to the hologram plane.

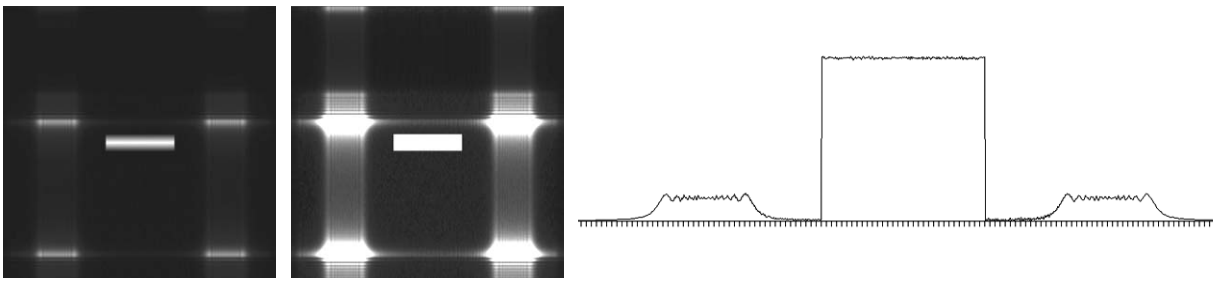


Fig. 5. Reconstructed image. The central peak in the graph is scaled down in order to visualize the small values of the distortions.

It follows from Figs. 2–4 that the distortions affect the amplitude reconstruction quality, but their influence can be minimized by choosing the distance to the hologram plane.

Figure 5 shows a reconstructed image with shift angles of 0, 10, 20, and 30° and a distance of 135 mm to the hologram plane. In the middle image, the intensity range is scaled down so that distortions are noticeable. On the right is a graph describing the amplitude distribution along the central line of the object. The distortion amplitude is less than 1%.

Thus, arbitrary shift angles can be selected when reconstructing images from mathematical holograms. This makes it possible to use random phase shift angle between the object and reference waves when recording holograms, thereby avoiding the need to use precision devices for introducing phase shifts in the optical setup. At the same time, there are distortions reducing the quality of the reconstructed image, but their impact may be neglected in most cases.

EXPERIMENTAL RESULTS

The experimental verification of the image focusing effects is carried out using a simple optical circuit (Fig. 6).

The anniversary silver badge of the Novosibirsk State Technical University was used as the object.

Figure 7 shows the results of interference between the reference and object beams with the varying phase shift angle.

The mathematical hologram (1) is formed from these patterns. The image is reconstructed using the Fresnel transform. The size of the object is 7 mm, the distance from the object to the image reconstruction plane is 135 mm.

The focusing effect of the holographic images, described in this work, was discovered by accident during an experiment. In one of the experiments, a device for controlling the piezoelectric ceramics shift was turned off. However, the image reconstruction results appeared to be comparable regardless of whether the device for introducing the phase shifts was turned on or turned off (Fig. 8)

A CanonEOS 650 SLR video camera was used to record digital holograms. The random shift was introduced by the oscillations of the device when the mirror on the camera was tilted as the hologram was recorded.

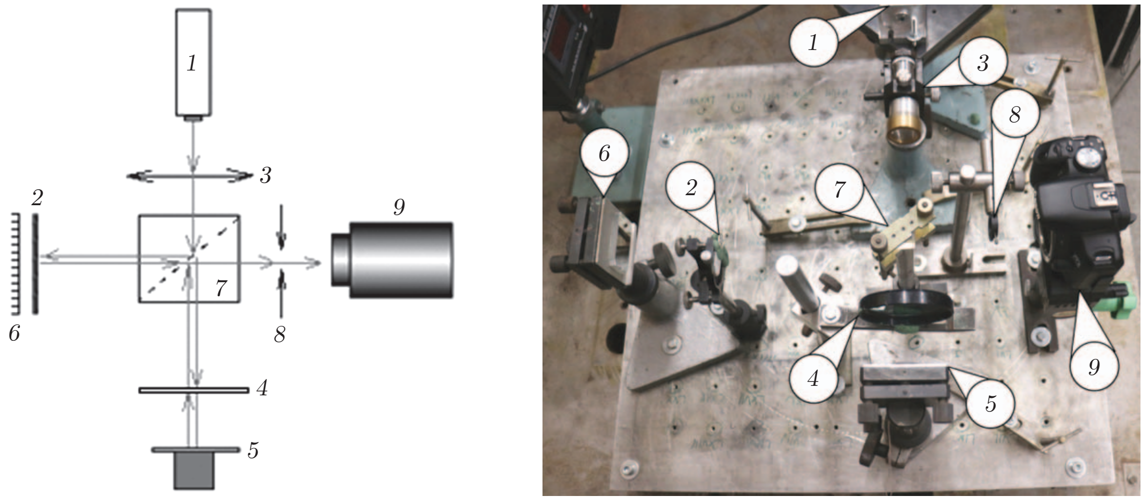


Fig. 6. Hologram recording scheme and its top view (laser *1*, object *2*, beam expander *3*, light filter for leveling the intensity *4*, reference mirror mounted on piezoelectric ceramics *5*, mirror for adjusting the device *6*, light beam divider *7*, diaphragm *8*, and video camera *9*).

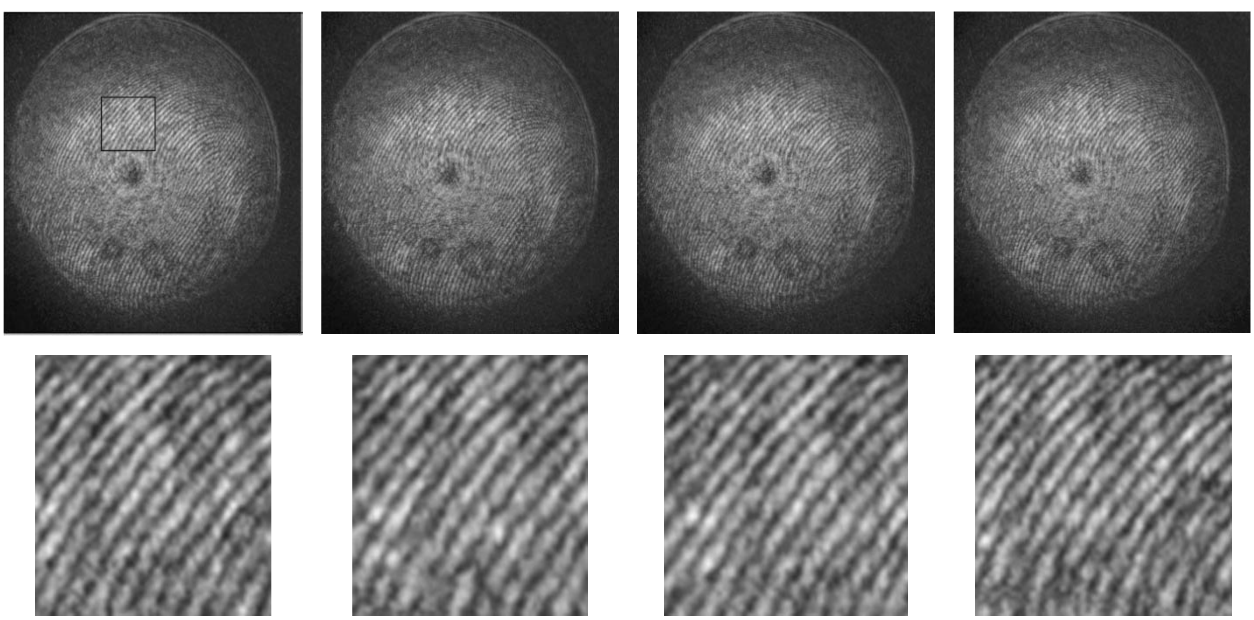


Fig. 7. Interference patterns with the varying phase shift angle (top row) and zoomed-in hologram fragments (bottom row).

Fig. 8. Results of the image reconstruction from the mathematical hologram (the original image of the badge is on the left, and the reconstructed field amplitude at random shifts is on the right).

CONCLUSIONS

This paper touched upon a new effect of image reconstruction from digital holograms obtained using the stepped phase shift method. It was shown that the distortions caused by incorrectly setting or using the erroneous values of the shifts during decoding slightly affected the image quality.

This effect allows one to create digital holographic systems with random phase shifts. These shifts can be generated by external noise, such as that occurring as the device vibrates, which makes it possible to reduce requirements for stabilizing the optical device and eliminate the use of precision devices for setting phase shifts.

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