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Expansion of Dynamic Range in Phase-Shifting interferometry

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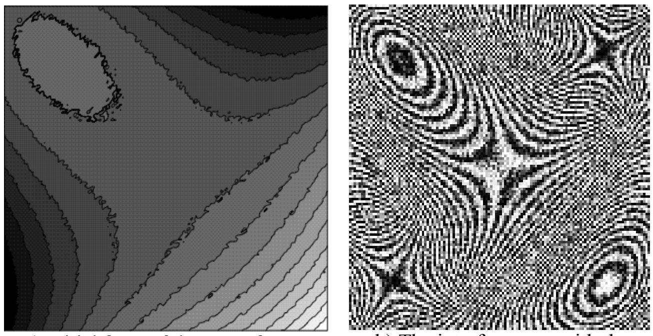
Abstract – **The paper considers a new method for measuring wave fronts with large curvature. The method is based on the analysis of the trajectory of interference signals (multidimensional Lissajous figures) formed by the intensity values of various points in a series of interferograms obtained by the stepwise phase shift method. The dynamic range is increased by forming the difference phases, similar to the methods of shear interferometry and solving the system of equations equivalent to the system of equations that is obtained when solving the Poisson equation and the Dirichlet boundary conditions. In contrast to classical methods of stepwise phase shift, the proposed method does not require the determination of actual values of phase shifts, which makes it possible to increase the accuracy of measurements and significantly expand the scope of this measurement method. The method is verified by measuring the wave front, which has a height much higher than the wavelength of the laser radiation.**

Index Terms – **phase shift, Lissajous curves, trajectory analysis.**

I. INTRODUCTION

Classical methods of an interferometry provide comparison of two wave fronts, one of which is reference. As a rule, the reference front represents a flat or spherical wave. In case of a difficult profile of a relief the shift of the wave front in adjacent points of an interferential picture exceeds the wavelength of a source of radiation and can't be measured unambiguously. In fig. 1 distribution of intensity is shown in the interferogram (fig. 1b) received with the flat basic wave front and the object wave front with big curvature (fig. 1a).



a) Initial form of the wave front b) The interferogram with the flat basic front

Fig. 1. Effect of spatial sampling of the field of brightness of the interferogram registered with the flat wave basic front at big curvature of the object wave front.

Here it is clearly visible that a considerable part of interferential strips spatially isn't allowed since a phase increment between the next points of the touch device of more λ, i.e. the optical difference of the course exceeds the wavelength of laser radiation. The following approaches are applied to expansion of dynamic range: shift interferometry [1] and method of equivalent wavelength [2]. These approaches are considerable complication of the optical scheme of the interferometer.

II. PROBLEM DEFINITION

The purpose of this work is development of a method of the analysis of interferograms with the step-by-step phase shift having the big dynamic range of change of a phase and not demanding obvious determination of size of the brought phase shifts. The essence of a method consists in formation of a difference of the phases between adjacent points of the interferogram which are similarly received in a method of a shift interferometry [3]

$$r_{ij} = \varphi_{ij} - \varphi_{i-1,j} + e_{ij}\,,\ c_{ij} = \varphi_{ij} - \varphi_{i,j-1} + h_{ij}\,,. \qquad (1)$$

where e_{ij} and h_{ij} noise component of an interferential signal.

III. DESCRIPTION OF A METHOD

The equation of intensivnost in each point of the interferogram at various sizes the brought phase shifts can be presented in the form [4]:

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$$\mathrm{I}_i(\mathrm{x},\mathrm{y}) = a(\mathrm{x},\mathrm{y}) + b(\mathrm{x},\mathrm{y})\cos\left[\phi(\mathrm{x},\mathrm{y}) + \delta_i\right], \qquad (2)$$

here $a(\mathrm{x},\mathrm{y})$ – average intensity, $b(\mathrm{x},\mathrm{y})$ – amplitude of interferential fringes.

It is possible to make assumption that in adjacent points (x, y) identical phase shifts δ_i are brought. This assumption is carried out in most cases proceeding from physical conditions of carrying out an experiment. Then we can receive the additional equations, considering decisions not in one, and in several spatial points $(\mathrm{x}_k, \mathrm{y}_k)$ of the interferogram.

$$\mathrm{I}_i(\mathrm{x}_k,\mathrm{y}_k) = a(\mathrm{x}_k,\mathrm{y}_k) + b(\mathrm{x}_k,\mathrm{y}_k)\cos\left[\phi(\mathrm{x}_k,\mathrm{y}_k) + \delta_i\right]. \qquad (3)$$

или в векторном виде

$$\vec{x}_i = \vec{x}_0 + \vec{b}\cos\left(\phi_i + \vec{\delta}\right), \qquad (4)$$

where i=0,1 ... N, N – number of samples, $\vec{\delta}$ – vector of phase shifts.

In the system of the equations (4) variables $\vec{x}_i$ can be considered as coordinates of points on the complex plane.

We will note that any point on the complex plane satisfying to the system of the equations (3) belongs to some spatial curve. So, for example, for 2D-planes is an ellipse well-known as Lissajous figure (addition of two sinusoidal waves of identical frequency) [5].

The ellipse equation corresponding to the system of the equations (4) for 2D-plane has an appearance

$$\frac{(x_1-(x_0)_1)^2}{b_1^2} + \frac{(x_2-(x_0)_2)^2}{b_2^2} - 2\frac{(x_1-(x_0)_1)(x_2-(x_0)_2)}{b_1 b_2}\cos\left(\phi_2-\phi_1\right) = \sin^2\left(\phi_2-\phi_1\right) \quad . \; . \qquad (5)$$

From (5) it is possible to define the difference of a phase between adjacent points $\varphi_{ij} = \phi_2 - \phi_1$ [6].

Generally we have the equation of a flat hyper ellipse of the hyper ellipsoid formed by crossing (6a) and hyperplanes (6b)

$$\vec{x}^T A \vec{x} = 0 \qquad (6a)$$

$$B\vec{x} = 0, \qquad (6б)$$

where A and B - matrix of coefficients of a hyper ellipse and vector of coefficients of the hyperplane respectively.

It is necessary to receive assessment φ_{ij} in each point on noisy phase differences. The task is redefined (i.e. the equations much more, than variables) therefore we use the method of the ordinary least squares (OLS) for finding of estimates of phases.

$$\varepsilon = \sum_{i=1}^{M}\sum_{j=1}^{N}[(r_{ij} - \varphi_{ij} + \varphi_{i-1,j})^2 + (c_{ij} - \varphi_{ij} + \varphi_{i,j-1})^2]. \qquad (7)$$

Differentiating (7) on φ_{ij} and equating results to zero, we will receive the following system of the equations:

$$4\varphi_{ij} - \varphi_{i-1,j} - \varphi_{i+1,j} - \varphi_{i,j-1} - \varphi_{i,j+1} = r_{ij} - r_{i+1,j} + c_{ij} - c_{i,j+1}, \qquad (8)$$

где $2 \le i \le M-1$; $2 \le j \le N-1$.

Zone approach consists in registration of phase differences in points according to geometry of splitting the touch device. Then the corresponding system of the normal equations of rather unknown values of phases which decides one way or another is determined by OLE. Until recently the given approach was used seldom. It has been caused by high labor input and low accuracy at the solution of system of the normal equations of big dimension.

In work [7] it is shown that at zone estimation of the image dimension the system of the normal equations by the form coincides with the system of the matrix equations:

$$\mathbf{A\Phi=V}, \qquad (9)$$

here **A** – Toeplitz matrix or diagonal-constant matrix $N^2 \times N^2$

$$A \approx \begin{bmatrix} A_0 & -I & & & & 0 \\ -I & A_0 & -I & & & \\ & -I & A_0 & -I & & \\ \hline & & -I & A_0 & -I & \\ & & & -I & A_0 & -I \\ 0 & & & & -I & A_0 \end{bmatrix}, \qquad (10)$$

where

$$A_0 = \begin{bmatrix} 4 & -1 & & & & 0 \\ -1 & 4 & -1 & & & \\ & -1 & 4 & -1 & & \\ \hline & & -1 & 4 & -1 & \\ & & & -1 & 4 & -1 \\ 0 & & & & -1 & 4 \end{bmatrix},$$

I – single matrix of the size $N \times N$, **Φ** - vector column $N^2 \times 1$ искомого распределения фазы; **V** - vector column $N^2 \times 1$ formed of values of differences of phases according to the system of the equations (2).

Such system of the equations is equivalent to the system of the equations which turns out at the solution of the equation of Poisson and boundary conditions of Dirichlet

$$\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} = 0, \; \varphi(0) = 0 \qquad (11)$$

on a rectangular grid when using standard five-pointed final and differential approximation [7]. The known effective direct algorithms of the solution of the equation of Poisson (11), for example, based on fast transformation of Fourier demand periodic continuation of the field of differences of phases that significantly narrows the field of their practical application, allowing to measure only rather simple symmetric wave fronts. However for an aperiodic case essential differences in boundary points take place and then the matrix (10) of system of the equations (9) takes a form

$$A=\begin{bmatrix} A_0 & -I & & & & 0\\ -I & A_1 & -I & & & \\ & -I & A_1 & -I & & \\ \hline & & -I & A_1 & -I & \\ & & & -I & A_1 & -I\\ 0 & & & & -I & A_0 \end{bmatrix}, \qquad (12)$$

where

$$A_0=\begin{bmatrix} 2 & -1 & & & & 0\\ -1 & 3 & -1 & & & \\ & -1 & 3 & -1 & & \\ \hline & & -1 & 3 & -1 & \\ & & & -1 & 3 & -1\\ 0 & & & & -1 & 2 \end{bmatrix} \text{ и}$$

$$A_1=\begin{bmatrix} 3 & -1 & & & & 0\\ -1 & 4 & -1 & & & \\ & -1 & 4 & -1 & & \\ \hline & & -1 & 4 & -1 & \\ & & & -1 & 4 & -1\\ 0 & & & & -1 & 3 \end{bmatrix}.$$

Note that the matrix a, in this case, has no Toeplitz form and the fast algorithm of the solution discussed above is not applicable.

At a small number of the equations for the decision (12) it is possible to use the known method of a matrix pro-race [8]. However in practical interferential measuring systems as the input equipment the photodetectors having 1024x1024 and more elements of permission are used. The method of a matrix pro-race demands storage in memory N vectors of the size N and N of matrixes of the NxN size that places too great demands on random access memory even for modern computer facilities.

Authors have offered a fast algorithm for a case when the matrix of A has various elements on the main diagonal. The system of the equations (9) can be factorized if there is a transformation such that

$$\mathbf{UAU}^T=\mathbf{\Lambda}, \qquad (13)$$

here $\mathbf{U}$ - transformation matrix, $\mathbf{\Lambda}$ - cell-diagonal matrix. From the theory of matrix calculations [3] it is known that if - the matrix, is made of own vectors of U_i of a matrix, and λ_i. her own numbers, then $\mathbf{\Lambda}$ – a diagonal matrix which elements are own numbers of λ_i.

$$\mathbf{\Lambda}=\begin{bmatrix} \lambda_1 & & 0\\ & \lambda_2 & \\ \cdots & \cdots & \cdots\\ 0 & & \lambda_N \end{bmatrix}. \qquad (14)$$

Then the matrix (4) will take the following form

$$\mathbf{A}=\begin{bmatrix} \Lambda_0 & -I & & & & 0\\ -I & \Lambda_1 & -I & & & \\ & -I & \Lambda_1 & -I & & \\ \hline & & -I & \Lambda_1 & -I & \\ & & & -I & \Lambda_1 & -I\\ 0 & & & & -I & \Lambda_0 \end{bmatrix}. \qquad (15)$$

By rearranging columns and rows, the matrix and elements of vector V are reduced to a tridiagonal form with dense diagonals. The system of equations (3) takes the form

$$\mathbf{UAU}^T\mathbf{\Phi}=(\mathbf{UV})^T. \qquad (16)$$

Eigenvalues $\lambda 0$ and $\lambda 1$ of the block matrices A_0 and A_1 are

$$\lambda 1_k=\sqrt{\frac{2}{N+1}}\cdot\left[4-2\cos\left(\frac{k\pi}{N+1}\right)\right],\ k\in[0,N-1],$$
$$\lambda 0_k=\lambda 1_k-1 \qquad (17)$$

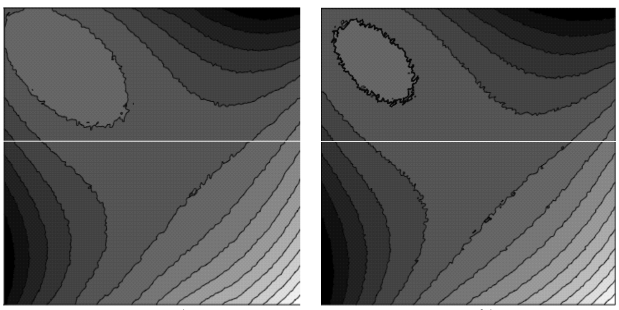
We obtain eigenvectors $\mathbf{U}$ by solving a system of equations

$$\mathbf{A}_1\mathbf{U}=\lambda 1_k\mathbf{A}, \qquad (18)$$

with their subsequent normalization. Note that eigenvectors $\mathbf{U}$ are the same for A_0 and A_1.

IV. Experimental Results

To verify the proposed algorithm, five interferograms with random phase shifts, similar to those shown in Fig. 1. In Fig. Figures 2 and 3 show the results of the phase calculation, according to the proposed algorithm. Figure 2 shows the shapes of the initial and reconstructed wave fronts, and Fig. 3 section of the wave fronts along their midline. From the obtained results it follows that the proposed approach makes it possible to measure the phase difference significantly exceeding the period of the interference fringes.



a) b)

Fig. 2. Initial wave front. (a) wave front (b) restored from interferograms.

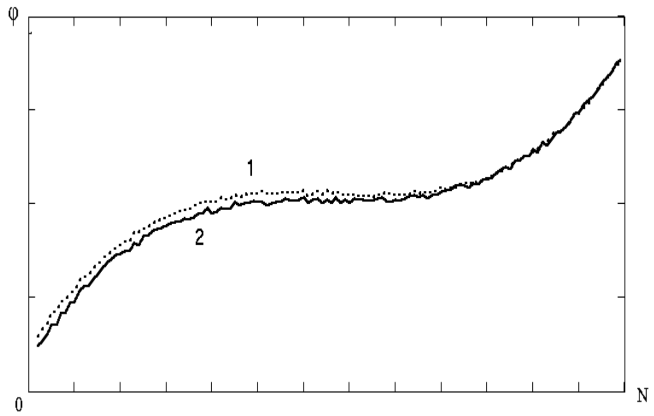


Fig. 3. Measurement results: 1-cross section of the restored phase field Fig. 2 (a), 2 – section of the phase test field Fig.2 (b).

IV. CONCLUSION

A new way of measuring wave fronts with large curvature is presented. The method is based on the solution of the Poisson equation under Dirichlet boundary conditions. The phase difference in adjacent points of the interferogram is computed according to the coefficients of the trajectory of interference signals (hyperellipse) formed by the insertion of phase shifts. The proposed approach does not require the determination of the introduced phase shifts.

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