The introducing of the feedback into optical measuring system for the increase of the accuracy parameters

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Annotation – A high-precision algorithm for detecting managing phase shits brought in optical systems is proposed, it is based on the interference figure intensity analysis in two points.

***Key words*: optical measurements, measuring systems, optical coherent systems, interferometry.**

I. Introduction

MIRROR FIXED ON PIEZOCERAMICS are commonly used for introduction managing influences into optical systems. However owing to phase shifters mistakes it is hard enough to identify exact value of entered phase shifts in practice. Therefore calibrating operations are needed before every set of measurements.

This paper examines a new method for random phase shifts exact detection via intensity set in two points of interference figure.

II. Problem statement

Interferogram accessing and decoding methods based on step-by-step shift became widely-spread lately when making a model of interference measuring systems (phase-sampling, phase-shifting interferometry) [1]. It is caused by simple definition of separate phase shift values, sufficiently easy algorithms and high precision decryption. Meanwhile existing interferometer schemes are common to modify. The single-step phase shift method is built on several interferogram registration while illuminating wave phase changing to known values. Phase measurement accuracy depends on the correctness of brought in phase shift value setting.

The point of the matter lies in entered phase shift real value definition by interference signal trajectory analysis in two random points (A and B) in interference pattern.

Interferogram intensity at the point *(x,y)* with phase shift δ_i with different phase shifts can be presented as

$$I_i = I_0(x,y)\left[1+V(x,y)\cos(\phi(x,y)+\delta_i)\right], \quad (1)$$

where $I_0(x,y)$ - average brightness, $V(x,y)$ - interference pattern visibility, $\phi(x,y)$ - phase difference of interfering wavefronts, i=0,2, ... , m-1, m – phase shifts number and $\delta_0 = 0$.

Usually decryption goal is to define phase difference of interfering wavefronts $\phi(x,y)$ by intensity values $I_i(x,y)$. Our task is to describe denotations δ_i.

III. Theory

One can make an assumption that phase shifts are equal in adjacent points. This assumption can be implemented in most cases in terms of experimental procedure physical conditions. Then we can get additional equations while examining solutions not at one, but at several spatial points (x_k, y_k).

$$I_{i,k} = I_{0,k}\left[1+V_k\cos(\phi_k+\delta_i)\right] \quad (2)$$

In the general case the number of points k = 1, . , n. General unknown quantity: n·3+m-1. Number of equations: n·(m-1). Solution can be found when general number of equations is AE unknown variable quantity, i.e.:

$$n\cdot m \geq 3n+(m-1). \quad (3)$$

Analytical solution can be found while registering 5 interferograms with phase shifts $\delta_0=0,\delta_1,\delta_2,\delta_3,\delta_4$. In this case we get 10 transcendental equations with 10 unknown variables ($I_{0,1}$, $I_{0,2}$, V_1, V_2, ϕ_1, ϕ_2, δ_1, δ_2, δ_3, δ_4). Solution can be found also for another number of points. This paper presents a numerical method of random phase shifts finding in intensity values at two accidental points of interference pattern.

$$I_{i,1} = I_{0,1}\left[1+V_1\cos(\phi_k+\delta_i)\right]$$
$$I_{i,2} = I_{0,2}\left[1+V_2\cos(\phi_k+\delta_i)\right] \quad (4)$$

where i=0,2, ... , m-1, m >= 5.

We continue searching the solution in complex plane with axes I_A, I_B where intensity values of first and second points corresponding to different phase shifts are set. A point describes some path at complex plane when changing shift angles from 0 to 2π (Fig.1). Interference signals trajectory in intensity space is the 2nd power of central curve – ellipse. One must describe equation coefficients of approximating curve to define the trajectory specifications [2].

$$a_{11}x^2 + a_{22}y^2 + 2a_{12}xy + 2a_{13}x + \\ + 2a_{23}y + a_{33} = 0 \qquad (5)$$

where x, y – intensity values at chosen points.

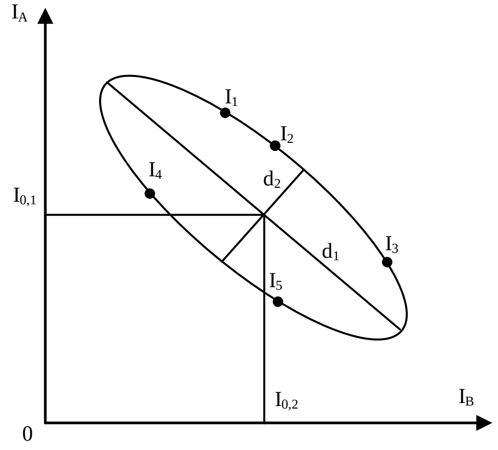

Fig. 1. Trajectory in complex plane where intensity values are placed in two chosen points of interferogram (d_1 , d_2 - main ellipse axes).

To retrieve coefficients in expression (5) interference signal path should be approximated by the 2nd power of two-dimensional polynomial. The usage of standart approach, based on least-squares method application can't provide required measurement inaccuracy. Therefore a stable method offered in [3] is elected for ellipse approximation. The method is based on solutions search, which satisfy the condition $4\,a_{11} \cdot a_{22} - a_{12} = 1$ marking out ellipse trajectory in other possible trajectory types which are the conic section (parabola, hyperbola). The given condition is probable to formulate as combined equations (6) [3]:

$$a^T C a = 1,$$

$$\text{Where } C = \begin{bmatrix} 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \qquad (6\text{-}1)$$

$$2S \cdot a - 2\lambda C a = 0 \qquad (6\text{-}2)$$

where $\vec{a}^T = [a_{11}, a_{22}, a_{12}, a_{13}, a_{23}, a_{33}]$,

$S = D^T D$,

$$D = \begin{bmatrix} x_1^2 & y_1^2 & x_1 y_1 & x_1 & y_1 & 1 \\ x_2^2 & y_2^2 & x_2 y_2 & x_2 & y_2 & 1 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ x_N^2 & y_N^2 & x_N y_N & x_N & y_N & 1 \end{bmatrix},$$

N - number of points used for trajectory approximation, and λ - matrix S (6x6 size) eigenvalues.

The given combined equations solution is the matrix S eigenvector s, which matches the same matrix minimal eigenvalue λ. Thus approximation problem reduces to generalized problem of eigenvectors and values.

Fig. 2 provides the interference signals trajectory in the intensity space. Points display the path with entered phase shifts, and the solid line is a result of its approximation by the 2nd power of polynomial (5).

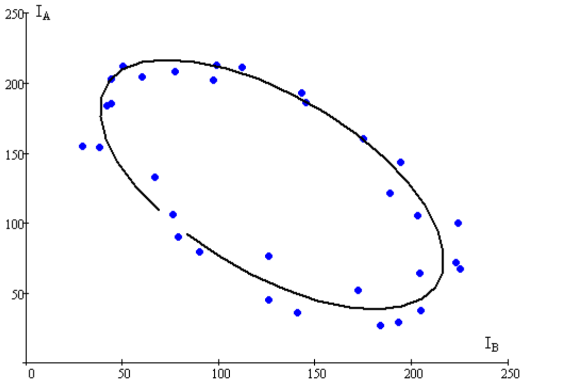

Fig. 2. The trajectory in a complex plane, which is circumscribed by real interference signals (solid line is the result of the 2nd power of polynomial approximation).

Average brightness levels match the center of ellipse coordinates and can be found like this

$$x0 = I_{01} = -\frac{\begin{vmatrix} a_{13} & a_{12} \\ a_{14} & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{vmatrix}},$$

$$y0 = I_{02} = -\frac{\begin{vmatrix} a_{11} & a_{13} \\ a_{12} & a_{14} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{vmatrix}} \qquad (7)$$

Phase shift angles may be directly detected when transforming the trajectory into circular path. To this effect the next reference quantity vector transformation is needed to be done:

a) to bring the center of ellipse to origin of coordinates

$$x1 = x - x0, \quad y1 = y - y0; \qquad (8)$$

b) to turn ellipse across one of the axis.

$$\begin{bmatrix} x2 \\ y2 \end{bmatrix} = \begin{bmatrix} \cos\Omega & \sin\Omega \\ -\sin\Omega & \cos\Omega \end{bmatrix} \begin{bmatrix} x1 \\ y1 \end{bmatrix} \qquad (9)$$

Rotation angle is specified by formula:

$$tg\Omega = \frac{2a_{12}}{a_{11} - a_{22}} \qquad (10)$$

c) ellipse stretch to the circle. Stretch factor γ determines from canonical ellipse equation, this ellipse is evaluated by its invariants

$$\lambda_0 x^2 + \lambda_1 y^2 + \frac{I_3}{I_2} = 0, \qquad (11)$$

Where $I_2 = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$, $I_3 = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$,

λ_0 and λ_1 - characteristic equation roots in

$$\begin{vmatrix} a_{11} - \lambda & a_{12} \\ a_{21} & a_{22} - \lambda \end{vmatrix} = 0.$$

Equation (11) shows that relation of characteristic equation roots equals the squares relation of ellipse diameters

$$\gamma = \sqrt{\frac{\lambda_0}{\lambda_1}}, \qquad (12)$$

Ellipse stretch in coordinate y performs like that

$$y2 = \frac{y2}{\gamma}.$$

Shift angles are determined by (circular) path coordinates this way:

$$\delta_i = arctg \frac{y2_i}{x2_i}. \qquad (13)$$

IV. EXPERIMENT OUTCOMES

Calculation results in model interference patterns are shown below. Number of phase shifts m=32. Fig. 3 displays an interference figure with 256 intensity levels with 2 chosen points and a trajectory built by 32 phase shifts. Phase shifts are random.

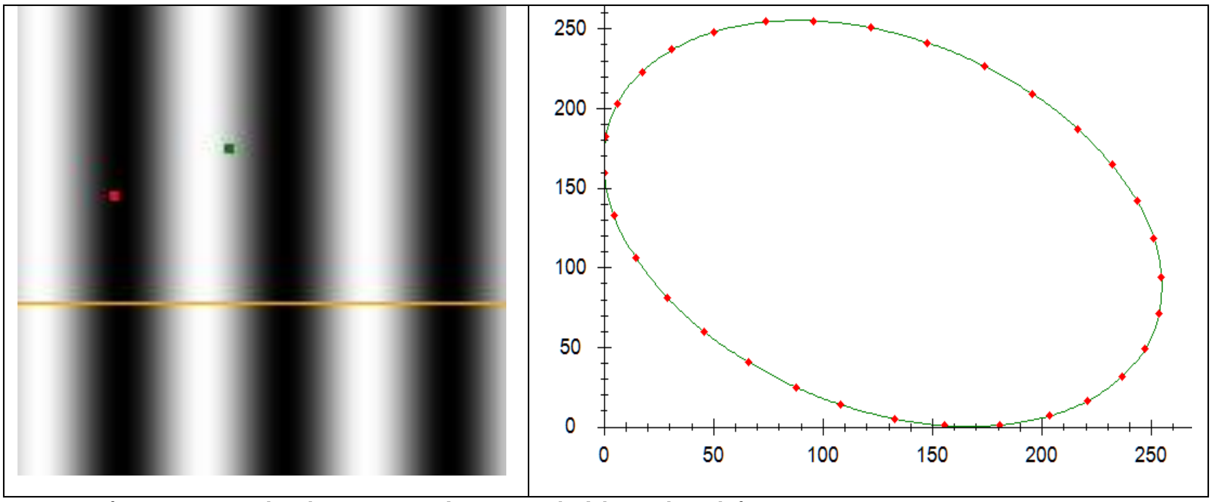

Figure 3. Interference pattern with 2 chosen points and a trajectory built by 32 phase shifts.

Diagrams of specified and received phase shifts are shown in fig. 4. Root-mean-square error is 0,00412 rad. Such a mistake is a result of discrete intensity definition. Intensity definition discrecity describes the overall accuracy which may be used to estimate phase shift angles.

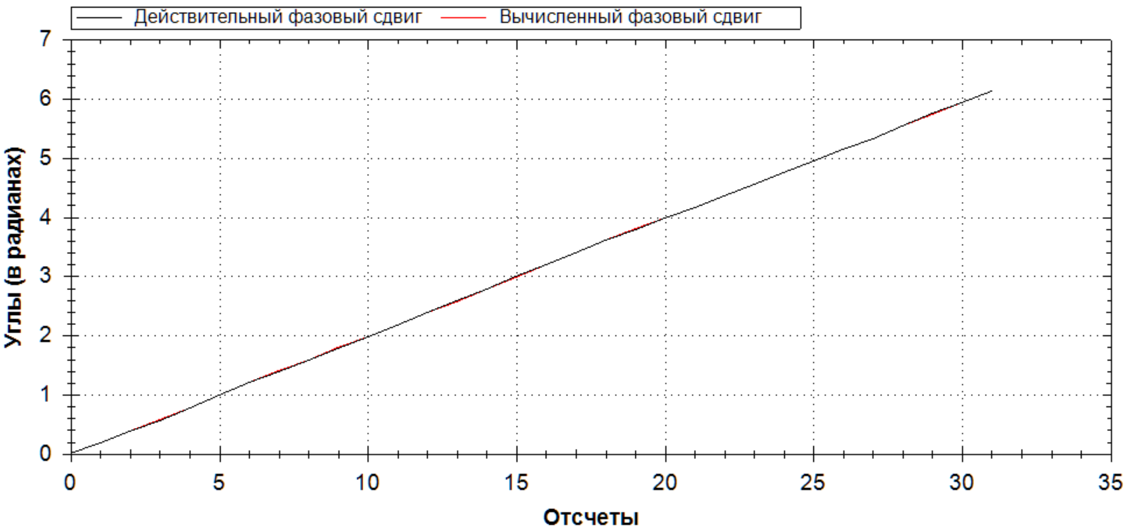

Fig. 4. Diagrams of specified and received phase shifts (32 shifts).

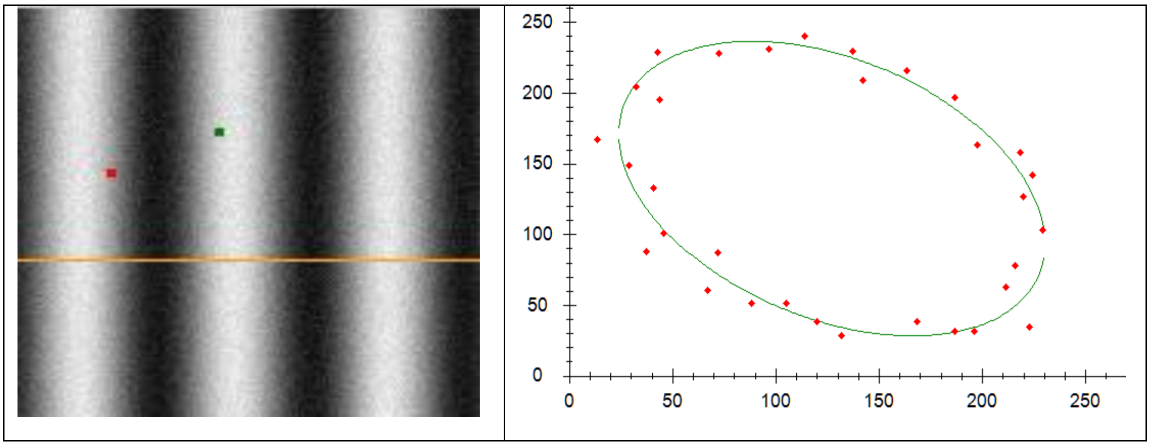

Fig5. Interference pattern with 10% mistake while determining the intensity and a trajectory built by 32 phase shifts.

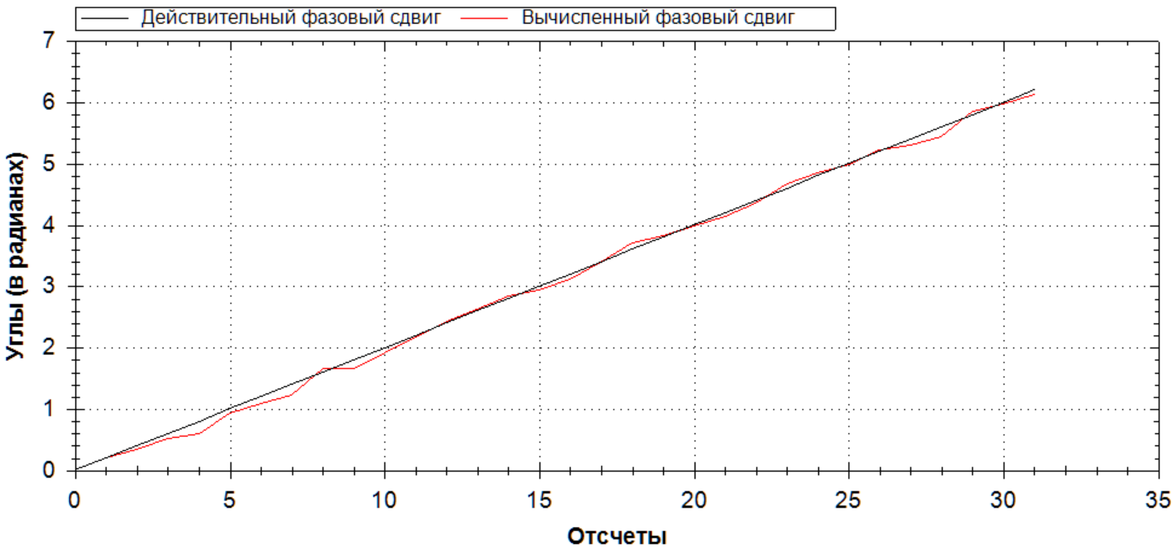

Figure 6. Diagrams of specified and received phase shifts with 10% mistake while determining the intensity.

Fig. 5 and 6 present the phase shift restoration results with 10% mistake while determining the intensity. The same number of shifts is set. In such case root-mean-square error of received shifts from specified shifts is 0,0665 rad.

V. Results discussion

Using step-by-step phase shift method, there are 2 main error types which cause the inaccuracy while determining phase difference – inaccuracy while setting up the phase shift and inaccuracy while intensity value calculation. Shift definition inaccuracy makes the largest contribution to inaccuracy at that [4,5].

The paper provides a new technique which gives the opportunity to calculate random phase shift angles by intensity values in two random points. Thereby intensity definition inaccuracy is the only inaccuracy type.

VI. Summary and conclusion

Effective method is developed for decoding the interference patterns, achieved by step-by-step phase shift technique. Such shift completely removes shift definition inaccuracy with the errors absence in intensity registration.

However errors relating to intensity measuring affect the phase shift definition accuracy. Digitizing error, which defines the method extreme exact characteristics, remains even in case of total noise lack in intensity registration.

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